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ON MULTIPLE REFLECTION WITHIN A SYMMETRICAL OPTICAL INSTRUMENT

By T. Smith

ABSTRACT

The Gaussian constants of a symmetrical optical instrument are built up from those of its parts by matrix multiplication. The matrix method is employed to find the constants for light which has been reflected any number of times between two given surfaces of the instrument. The matrix is expressible as the sum of two parts, each consisting of the product of a trigonometrical function depending on the number of reflections and a matrix which is independent of this number.

In a paper in the February number of the Journal of the Optical Society of America Professor Southall has dealt with the determination of the cardinal points of a single lens for light twice reflected within the lens. I propose in this note to deal with the more general case when the number of reflections between any given pair of surfaces in a centered instrument is arbitrary. In discussing this problem I shall employ simple matrices; the matrix is decidedly the most suitable tool to employ when the general properties of the first order of any optical instrument are involved, as I think the following discussion clearly illustrates. For readers who are not familiar with the properties of matrices it will suffice to say that a matrix is a collection of numbers which has certain properties enabling it to be used to some extent as though it were a single algebraic quantity. Each number in a matrix is identified by the position it occupies when the matrix is written out at length. Matrices can be added by adding corresponding elements; thus if

$$\begin{pmatrix} B & D \\ A & C \end{pmatrix} = \begin{pmatrix} b_1 & d_1 \\ a_1 & c_1 \end{pmatrix} + \begin{pmatrix} b_2 & d_2 \\ a_2 & c_2 \end{pmatrix},$$

each set of four quantities inside brackets is a matrix, and the one equation implies the four algebraic equations

$$A=a_1+a_2, \quad B=b_1+b_2,$$
$$C=c_1+c_2, \quad D=d_1+d_2.$$

If we agree to use a convention by which the three matrices written at length above are denoted by (W), (w_1) and (w_2), respectively, we secure distinct economy of space by writing the relation in the form

$$(W)=(w_1)+(w_2)$$

in place of the four equivalent relations. The importance of matrices arises however chiefly from the laws which govern their multiplication. Keeping to the definitions of (W), (w_1) and (w_2) that have just been given, the equation

$$(W)=(w_1)(w_2),$$

in which the order of the factors (w_1) *and* (w_2) *is important*, implies that the constituents of (W) may be calculated by the equations

$$A=a_1b_2+c_1a_2, \quad B=b_1b_2+d_1a_2,$$
$$C=a_1d_2+c_1c_2, \quad D=b_1d_2+d_1c_2,$$

the constituents of the expression representing the value of any element in (W) being taken from the corresponding row of (w_1), the matrix written on the left of the product, and the corresponding column of (w_2), the matrix on the right. Of course the first element of the row is multiplied into the first element of the column, and the second elements are similarly to be taken together. The importance of the order may be seen by interchanging the factors. Thus applying the rule just given

$$\begin{pmatrix} b_2 & d_2 \\ a_2 & c_2 \end{pmatrix}\begin{pmatrix} b_1 & d_1 \\ a_1 & c_1 \end{pmatrix}=\begin{pmatrix} b_1b_2+a_1d_2 & d_1b_2+c_1d_2 \\ b_1a_2+a_1c_2 & d_1a_2+c_1c_2 \end{pmatrix},$$

all the elements differing from those of the previous product. It follows that the order of the factors in a product may not in general be changed, but in a continued product such as $(w_1)(w_2)(w_3)$ we may either first find the product $(w_1)(w_2)$ and multiply this by (w_3) as a following matrix or first find $(w_2)(w_3)$ and then use this product as a matrix to follow (w_1). In other words the order of the factors matters but not the order in which arithmetical operations are carried out.

Now Gauss showed that the first order properties of any symmetrical optical instrument are determined by four quantities a, b, c, d, of which a represents the power of the system, b and c represent pure numbers

depending on the points chosen in the object space and the image space respectively as the origins of coordinates—b and c in fact are the transverse magnifications for these particular points—and d is a length, $-d$ representing the reduced optical separation between the two origins. If x and x' are the reduced distances of a pair of conjugate points on the axis each from its appropriate origin, the relation between x and x' is

$$axx' + bx' - cx - d = 0 \tag{1}$$

where both x and x' are measured from the origins in the same direction, viz., the direction in which light travels from the object space to the image space. Now if we have two systems, distinguished by the suffixes 1 and 2, which are to be combined into a single system, we may eliminate x' between the two equations

$$a_1xx' + b_1x' - c_1x - d_1 = 0$$

$$a_2x'x'' + b_2x'' - c_2x' - d_2 = 0$$

thus:

$$\frac{c_1x + d_1}{a_1x + b_1} = x' = -\frac{b_2x'' - d_2}{a_2x'' - c_2}$$

or

$$Axx'' + Bx'' - Cx - D = 0$$

where A, B, C, D are the elements of the matrix (W) obtained from the equation

$$(W) = (w_1)(w_2).$$

In other words, the combination of two optical systems to form a compound instrument is represented in the Gaussian theory by the multiplication of the component matrices to yield the matrix of the combination.

From (1) it is obvious that if we turn the instrument round so that object and image spaces are interchanged we have simply to consider b and c interchanged in their places in the matrix. If the system consists of a single surface with both reference points at the vertex we obtain the well known equation by putting $b = c = 1$, $d = 0$. That is to say the matrix for a single surface of power a is

$$\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$$

provided the reference points are coincident at the vertex of the surface. If we combine a number of surfaces having a common vertex we obtain for the matrix of the complete system

$$\begin{pmatrix}1 & 0\\ a_1 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ a_2 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ a_3 & 1\end{pmatrix}\cdots=\begin{pmatrix}1 & 0\\ a_1+a_2+a_3+\cdots & 1\end{pmatrix}$$

on applying the multiplication rule, illustrating that results usually obtained in another way are merely special cases of matrix multiplication. The reader will easily verify for himself that a shift of origin in the object space through the reduced distance X is equivalent to prefixing the matrix

$$\begin{pmatrix}1 & -X\\ 0 & 1\end{pmatrix},$$

and similarly a movement in the image space through the reduced distance X' is represented by postfixing the matrix

$$\begin{pmatrix}1 & -X'\\ 0 & 1\end{pmatrix}.$$

Thus the matrix for a single lens having surfaces of powers a_1 and a_2 and reduced thickness t, with the origins at the vertices, is

$$\begin{pmatrix}1 & 0\\ a_1 & 1\end{pmatrix}\begin{pmatrix}1 & -t\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ a_2 & 1\end{pmatrix}=\begin{pmatrix}1-a_2t & -t\\ a_1+a_2-a_1a_2t & 1-a_1t\end{pmatrix}.$$

We are now in a position to deal quite generally with the problem of reflection within a refracting instrument, for we are entitled to regard reflection as a special case of refraction.[1] Let us suppose that the whole system consists of m surfaces 1, 2, 3 $\cdots$ m and that reflection takes place at surfaces i and j, the light travelling n times between the two before being refracted out of the system. Regard the system as consisting of three parts, the points of division being at surfaces i and j. We consider these surfaces themselves divided, as regards their refracting power, into two parts. If the refractive indices on the object and image sides of surface i are μ_i and μ_i' respectively, and the curvature is R_i, we represent this surface in the first portion of the system by the matrix

$$\begin{pmatrix}1 & 0\\ -\mu_iR_i & 1\end{pmatrix}.$$

[1] For a detailed discussion of this question with special reference to the signs of all the quantities involved see *The treatment of reflection as a special case of refraction, Trans. Opt. Soc., 27*, p. 312; 1925–26.

The contribution of this surface to the second part of the system is

$$\begin{pmatrix} 1 & 0 \\ \mu_i' R_i & 1 \end{pmatrix},$$

and surface j is treated analogously. Denoting the three matrices by (w_1), (w_2), (w_3) respectively, and adding an accent when the b and c are interchanged through light travelling in the reverse direction and therefore encountering the surfaces in the reverse order, we obtain for light reflected an even number of times the matrix product

$$(w_1)(w_2)(w_2')(w_2)(w_2') \cdots (w_2)(w_3)$$

and for light reflected an odd number of times the product

$$(w_1)(w_2)(w_2')(w_2)(w_2') \cdots (w_2')(w_1')$$

the number of matrices with the suffix 2 present in either product being the number of times the light has travelled between surfaces i and j, that is to say it is greater by one than the number of reflections that take place. Now in the Gaussian theory the four coefficients a, b, c, d of any system are not independent, but satisfy the condition $bc-ad=1$. We may therefore represent any matrix by the expression

$$\begin{pmatrix} p\cos\theta & -q\sin\theta \\ \frac{1}{q}\sin\theta & \frac{1}{p}\cos\theta \end{pmatrix} \tag{2}$$

The multiplication law gives

$$\begin{pmatrix} p\cos\theta & -q\sin\theta \\ \frac{1}{q}\sin\theta & \frac{1}{p}\cos\theta \end{pmatrix} \begin{pmatrix} \frac{r}{p}\cos\phi & -\frac{a}{r}\sin\phi \\ \frac{r}{q}\sin\phi & \frac{p}{r}\cos\phi \end{pmatrix}$$

$$= \begin{pmatrix} r\cos(\theta+\phi) & -\frac{pq}{r}\sin(\theta+\phi) \\ \frac{r}{pq}\sin(\theta+\phi) & \frac{1}{r}\cos(\theta+\phi) \end{pmatrix}$$

and this formula is immediately applicable to the repeated matrices with suffix 2. Evidently if (2) represents the matrix (w_2), the middle system after $n-1$ reflections is represented by the matrix

$$\begin{pmatrix} r\cos n\theta & -\dfrac{pq}{r}\sin n\theta \\ \dfrac{r}{pq}\sin n\theta & \dfrac{1}{r}\cos n\theta \end{pmatrix}$$

where $r=1$ if n is even and $r=p$ if n is odd. We can therefore express the resultant matrix after any number of reflections as the product of three matrices.

A number of interesting forms are obtained by regarding these cosines and sines in a suitable way. For example, if n is even we may write

$$(W)=-\frac{\sin(n-2)\theta}{\sin 2\theta}(W_i)+\frac{\sin n\theta}{\sin 2\theta}(W_j)$$

where (W) is the matrix for light that has passed n times between surfaces i and j, and (W_i) and (W_j) are those for light which has travelled directly to surfaces i and j, respectively, and after a single reflection there has been refracted out of the instrument after regaining the first surface. Similarly if n is odd we have

$$(W)=-\frac{\sin(n-3)\theta}{\sin 2\theta}(W_1)+\frac{\sin(n-1)\theta}{\sin 2\theta}(W_3)$$

where (W_1) and (W_3) are the matrices for light that has been refracted straight through the system and for light reflected once at each of the surfaces i and j respectively. When these equations are used θ may be found from $\sin 2\theta=2\sqrt{-abcd}$, or if one of the expansions for $\sin n\theta/\sin\theta$ in terms of $\cos\theta$ is employed we may use $\cos 2\theta=bc+ad=2ad+1$. Obviously when a, b, c, d are all of the same sign hyperbolic sines and cosines will be used.

The quantities p and q are not calculated in practice—they were merely introduced for purposes of demonstration. When the number of reflections is odd we need only trace as far as a reflecting surface, and from the values found for a, b, c, d calculate $2ac$, $2bd$, $2ad+1$; when it is even we can put the central contribution in the form

$$\cos(n-1)\theta\begin{pmatrix} b_2 & d_2 \\ a_2 & c_2 \end{pmatrix}+2a_2b_2c_2d_2\frac{\sin(n-1)\theta}{\sin 2\theta}\begin{pmatrix} \dfrac{1}{c_2} & \dfrac{1}{a_2} \\ \dfrac{1}{d_2} & \dfrac{1}{b_2} \end{pmatrix}$$

which is convertible immediately to the form given above on writing $\{\sin(n-1)\theta\cos 2\theta - \sin(n-3)\theta\}/\sin 2\theta$ for $\cos(n-1)\theta$. The most interesting form for the central contribution with an even number of reflections is

$$\frac{\sin(n+1)\theta}{\sin 2\theta}\begin{pmatrix} b_2 & d_2 \\ a_2 & c_2 \end{pmatrix} - \frac{\sin(n-1)\theta}{\sin 2\theta}\begin{pmatrix} b_2 & -d_2 \\ -a_2 & c_2 \end{pmatrix},$$

which shows that we may consider the light to have traversed this part of the system a negative number of times, the power of each surface being then supposed changed in sign, the separations of the surfaces also being negative.

The actual problem investigated here is doubtless trivial from a practical point of view, but the theory given is applicable to a number of problems which are by no means trivial. For example we can write down at once the constants of a system in which the same symmetrical parts are repeated any number of times.

THE NATIONAL PHYSICAL LABORATORY,
TEDDINGTON, ENGLAND.
AUGUST 9, 1928.

THE USE OF CRYSTALLINE QUARTZ FOR SPECTROGRAPHIC WORK

By Joseph W. Ellis

While studying the infrared absorption of a dilute solution of an organic crystalline solid in a solvent very transparent to infrared rays, namely benzophenone in carbon tetrachloride, some features appeared in the spectrum which at first were mistaken for shallow absorption bands. These details were brought into evidence only through the fact that the spectrum was automatically recorded on a photographic plate. Such a record is shown in Fig. 1A. The bands marked *a*, *b*, *c*, and *d* are characteristic of the solute and will be discussed elsewhere. The band marked *e*, and the corresponding band in all of the following records, is the 1.39μ absorption maximum of a 2 mm cell of chloroform, which is used as a standard of reference. As a result of

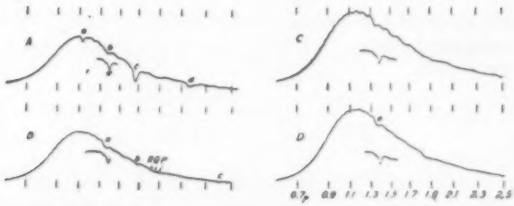

Fig. 1. *A. Absorption record: benzophenone in carbon tetrachloride, showing spurious minima. B. Absorption record: carbon tetrachloride (with an impurity). C. Spurious effects with quartz end-plate only; transmission planes 45° to vertical. D. Spurious effects with quartz end-plate only; transmission planes vertical and horizontal.*

a study of a large number of organic liquids, the minima in the curve between 1 and 1.5μ were somewhat unexpected. They were nevertheless assumed to be true absorption bands.

The absorption cell, 40 mm long, was then filled with carbon tetrachloride alone (E. K. Co. No. 444) and the bands persisted, as shown in Fig. 1B. In addition small bands, *a*, *b*, and *c* appeared. These are probably due to the presence of a small amount of impurity, possibly chloroform, in the carbon tetrachloride. *P*, *Q* and *R* are the negative, zero and positive branches of the well-known 1.87μ atmospheric water-vapor band.

Quartz end-plates had been used on the absorption cell and the question arose as to whether these were introducing spurious effects.

They were examined under crossed polarizing prisms and one of the two was found to yield the familiar rings and brushes indicating that its optic axis was perpendicular to the faces; the other one restored white light at angular positions 90° apart, indicating that the optic axis was parallel, or nearly parallel, to the surfaces. This was rather surprising inasmuch as this pair of end-plates had been purchased specifically for absorption cell work.

The rather regularly recurring minima of Figs. 1A and 1B were then suspected of being produced by some polarization phenomenon in this latter quartz plate. Its vibration planes were located and it was placed before the slit of the spectrograph with these planes making 45° angles with the vertical. The effect again appeared, Fig. 1C.

From a study of the optical system, shown in Fig. 2A, the following seems to be a partial solution of the appearance of the transmission minima. Light from the source F becomes partially plane polarized

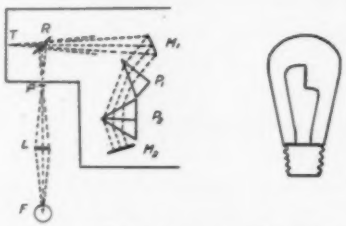

FIG. 2. *A. The optical system. B. The lamp source (filament and wall non-parallel).*

in a horizontal plane because of reflection losses at the surfaces of the surrounding bulb, Fig. 2B, and the quartz lens L. This effect is enhanced by the finite height of the source, which is a single vertical helical filament, 12 mm×1 mm. The quartz plate P, 2.43 mm thick, oriented as indicated above, acts alternately as multiple whole wave-plate and odd multiple half wave-plate for successive regions of the spectrum. For the sake of brevity we will refer to the wave lengths of such regions by the symbols $\lambda_{2\tau}$ and λ_{τ} respectively. The beam of light enters the spectrograph, is reflected from the 45° silver-on-glass reflector R and then by the parabolic mirror M_1 into the prism train. The prism system consists of one 30° and one 60° quartz prism, the latter of the Cornu type. Light passes a second time through this pair of prisms after reflection from the plane mirror M_2. That portion of the rays of the $\lambda_{2\tau}$ type will traverse the prisms with a greater loss of intensity than that portion of the λ_{τ} type. For there are three oblique air-quartz

boundaries and three corresponding quartz-air boundaries to be penetrated, and the λ_r rays will pass through vibrating parallel to the plane of incidence. Calculation by means of Fresnel's equations shows that for rays going through at minimum deviation there will be a 220 percent better transmission for one type of vibration than for the other. Thus as M_2 rotates, causing the spectrum to pass over the receiving thermopile T, there will be superimposed upon the regular distribution of energy due to the unpolarized portion of the light from F maxima and minima, such as seen in Figs. 1A-1C, due to that portion initially polarized in a horizontal plane. These curves have been traced by a beam of light deflected from a high-sensitivity galvanometer actuated by the thermopile T. A fuller description of this recording device has already been given in this journal.[1]

If the above analysis of the occurrence of maxima and minima is correct the effect might be expected to disappear if the transmission planes of the plate were placed in vertical and horizontal positions, in contrast to the 45° positions of Fig. 1C. A record was taken in this manner, Fig. 1D, but although the effect is diminished, it still persists suggesting that some other factor has not yet been considered. In this record also the 1.37μ atmospheric water-vapor band, marked a, appears as well as the one at 1.87μ.

When a piece of plane parallel glass was placed between F and L, oriented so that the rays would strike near the polarizing angle and in such a way as to have the plane of incidence horizontal, apparently just enough light was plane polarized vertically to annul the effect of the original horizontally polarized portion. This would superpose a set of minima and maxima upon the original maxima and minima respectively, thus once more smoothing out the energy curve. Such a record is shown in Fig. 3A. The transmission planes of the quartz plate were in the 45° positions.

Again, when the glass was rotated around so that the plane of incidence was vertical the two polarization effects added together, enhancing the maxima and minima as shown in Fig. 3B. Fig. 3C contains a record taken under conditions similar to those in Fig. 3B but with the glass plate between L and P.

The measurements of wave lengths and plate thickness are sufficiently accurate to permit a quantitative check on the theory presented. Suppose, for example, that the wave-length determinations λ are made for

[1] Ellis, J.O.S.A. & R.S.I., *11*, pp. 647–652; 1925.

several of the transmission minima of the curve. When these are substituted into the formula

$$\frac{(\mu_e-\mu_0)t}{\lambda}=n \qquad (1)$$

n should come out with integer values. Here $(\mu_e-\mu_0)$ is the maximum difference between the indices of refraction for the extraordinary and ordinary rays, and t is the plate thickness. Following is a sample

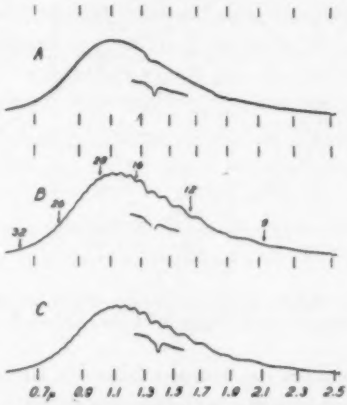

FIG. 3. *A. Polarization effect annulled by introduction of glass plate. B and C. Polarization effect enhanced by glass plate.*

substitution and a table which show the values of n for several bands indicated in Fig. 3B. λ and t are expressed in micra. The values for $(\mu_e-\mu_0)$ vary with wave length but may be obtained from curve *A* of Fig. 4, which has been plotted from Landolt-Börnstein tables.

$$n=\frac{0.00840\times 2430}{1.66}=12.3$$

TABLE 1. *Integer values (n) of multiple wave-plate.*

λ	n Eq. (1)	n (true)	% difference
2.14μ	9.2	9	2.2
1.66	12.3	12	2.5
1.27	16.5	16	3.1
1.04	20.6	20	3.0
0.805	26.9	26	3.5
0.665	32.9	32	2.8

It is observed that the values of n differ from whole numbers by amounts which increase with the numbers. That the theory so far advanced is essentially correct is evident from the approximate agree-

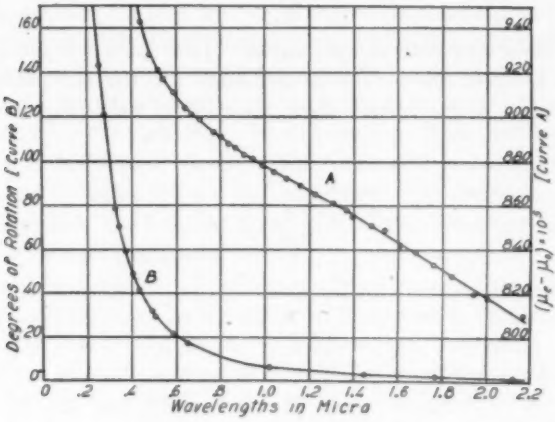

FIG. 4. *Curve A. Difference between refractive indices for extraordinary and ordinary rays in quartz. Curve B. Rotary dispersion of quartz.*

ment between $\Delta\lambda$ values determined between successive minima and the values determined by differentiation of Eq. (1):

$$\Delta\lambda = -\frac{\lambda^2 \Delta n}{(\mu_e - \mu_0)t} = \frac{\lambda^2}{(\mu_0 - \mu_e)}. \qquad (2)$$

A number of $\Delta\lambda$ values are tabulated in Table 2. These are independent of any assignment of value to n. Furthermore, the λ^2 term of Eq. (2) explains the progressive increase in $\Delta\lambda$ with longer wave lengths.

TABLE 2. *Wave length differences* ($\Delta\lambda$) *between adjacent minima.*

λ	$\Delta\lambda$ observed	$\Delta\lambda$ Eq. (2)
2.24μ	0.240μ	0.260μ
1.60	0.135	0.125
1.23	0.065	0.072
0.92	0.040	0.040
0.74	0.020	0.025

The approximate constancy in the percent difference column of Table 1 suggests that either t or $(\mu_e - \mu_0)$ of Eq. (1) must be incorrectly

determined. The former possibility is eliminated since t was constant within 1/3 percent. The tabulated values of $(\mu_e-\mu_0)$ are maximum values and hold only for perpendicular incidence upon a plate cut with the optic axis exactly in the surfaces. Calculation shows that the greatest deviation from the maximum value due to the non-parallel nature of the beam of light would not exceed one percent. The only explanation, therefore, for the larger deviations of Table 1 is a possible slight departure from parallelism between the optic axis and the surfaces of the plate.

The lens L, and the prism P_1 during the initial passage of the light through this prism, both rotate the plane of polarization. The effective thicknesses of these are 5 mm and 6 mm respectively. In Fig. 5 let RR' and QQ' represent the transmission planes of the quartz plate and WW' the direction of vibration of the initially polarized waves. The long waves of the λ_{2r} and λ_r types will emerge from the plate with

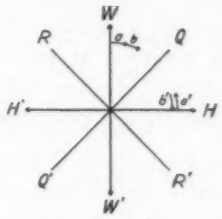

FIG. 5.

vibrations along WW' and HH' respectively. Because of the increasing amount of rotation of the plane of polarization with decreasing wave lengths (curve B, Fig. 4) the planes of vibration of the shorter wave lengths of the λ_{2r} and λ_r types will approach the common plane QQ' upon emergence. This is represented by arrows a and a' on Fig. 5. If the effective 6 mm of quartz in P_1 rotate the plane of polarization in the same direction that L does, then the rotation by P_1 will approximately annul the counter-clockwise rotation of the λ_r vibrations and will increase the rotation of the λ_{2r} vibrations. This is represented by the arrows b' and b. Finally, when the total effective 11 mm of quartz has rotated the plane of the latter type through 90° the variation of intensities between maxima and minima in the energy curve should go through a pronounced minimum. Curve B of Fig. 4 shows that this should occur near 1μ. The curves of Figs. 3B and 3C show this minimum variation at 1μ. The occurrence of this effect is not dependent

upon the assumption that the quartz of L and P_1 are either both right-handed or both left-handed.

The appearance of bands in the record of Fig. 1D can also be explained through the rotation of the plane of polarization by the lens.

Through the use of properly cut end-plates all of the spurious effects described above will disappear with the exception of the rotation of the plane of polarization. In the infrared region of the spectrum this effect alone would modify the distribution of energy only by the introduction of broad widely separated maxima and minima. Because of the rapid rise of curve B, Fig. 4, an analogous effect in the ultraviolet region would introduce relatively closely spaced maxima and minima. Thus, if due to any cause a certain amount of plane-polarization was introduced into a light beam at or near the source and the light then penetrated several millimeters of crystalline quartz along the optic axis before entering a prism train, considerable error might be made in measuring relative intensities throughout the ultraviolet.

The effects displayed in Figs. 1 and 3 recall a similar one produced through the use of mica windows at slightly longer wave lengths, described by Meyer and Bronk.[2] In that instance, however, the effect was one of interference rather than of polarization.

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[2] Meyer and Bronk, Astrophys, Jl., *59*, pp. 252–258; 1924.

INSTRUMENT SECTION

SOME PROPERTIES OF A FUSED SILICA TUNING-FORK

By Elmer A. Harrington

ABSTRACT

The logarithmic decrement, the coefficient of damping, and the coefficient of stiffness of a fused silica tuning-fork were determined from data obtained from photographic records of the vibrations of the tuning-fork after it had been struck with a pianoforte hammer.

Several years ago the late Professor A. G. Webster exhibited a fused silica tuning-fork at a meeting of the National Academy of Sciences which aroused considerable interest among the physicists who examined it. Apparently very few fused silica tuning-forks have ever been made, and a review of the literature has failed to show any data on them. It seems desirable, therefore, to publish the observations which were obtained for the purpose of calculating some of the physical properties of Professor Webster's tuning-fork, even after this lapse of time.

APPARATUS AND METHOD OF EXPERIMENT

The tuning-fork was made by bending a rod of fused silica about 4 mm in diameter, and attaching a stem, as in glass work. The prongs were about 10 cm long and 2 cm apart, and the stem consisted of a piece of the same rod about 4 cm long fused to the fork at the bend. It was constructed in the laboratories of the General Electric Company at Lynn under the direction of Professor Elihu Thomson, as a result of exhaustive investigations which showed fused silica to be "admirably adapted to the making of tuning-forks"[1] because of its high elasticity and low coefficient of expansion.

For purposes of observation the axis of the tuning-fork and the plane of the prongs were kept horizontal by clamping the stem between two pieces of soft wood in a heavy vise. A small thin mirror, 2 mm square, was stuck to the end of one prong in order to obtain a permanent record of the vibrations. The tuning-fork was then balanced by adding the proper amount of wax to one of the prongs. A fine beam of light from an arc lamp was reflected from the mirror to a photographic film on a revolving drum a meter in circumference, mounted in a light-tight box placed two meters away from the tuning-fork. By pressing a push-button switch, the shutter of the box could be opened and held open for any desired length of time. The speed of the drum was such that one hundred complete vibrations of the tuning-fork were recorded on a strip of film about 10 cm long.

[1] Private communication to the author.

The method of recording photographically the vibrations of the tuning-fork was briefly as follows: The drum was rotated at the desired speed by an electric motor, and then the tuning-fork was set in vibration by striking it a light blow with a pianoforte hammer. The harmonics and forced vibrations set up by the hammer were found to be negligible in about two seconds, after which the shutter of the drum-box was opened for about 3 seconds to record the damped vibrations of the tuning-fork.

The effect of damping was investigated by dividing the vibrations, as recorded on the photographic film, into groups of 50 each and measuring the amplitude of each succeeding 50th vibration. The data given in Table 1 were obtained from such a record.

TABLE 1. *Number of vibrations and corresponding amplitudes.*

Number of vibrations N	Amplitudes of vibrations on film (cm)	Natural logarithms of amplitudes Y
0	1.966	0.6762
50	1.928	0.6566
100	1.886	0.6345
150	1.858	0.6197
200	1.830	0.6045
250	1.798	0.5868
300	1.760	0.5654
350	1.732	0.5495
400	1.702	0.5320
450	1.670	0.5140
500	1.640	0.4961
550	1.604	0.4726
600	1.580	0.4576
650	1.556	0.4422
700	1.530	0.4254

THEORETICAL CONSIDERATIONS

A tuning-fork may be regarded as an instrument to which an amount of energy is imparted at one instant, and which is then left to itself. In addition to producing sound waves and to communicating some motion to the support, a tuning-fork is always subject to internal dissipative forces, the absence of which is never more than approximated to in practice. There is consequently a loss of energy due to generation of heat at points in the material during a vibration. The vibrations therefore must decrease in amplitude, since they depend for

their maintenance upon the initial store of energy which suffers gradual exhaustion. But, however large a fraction of the whole the loss of energy due to this cause may be, it is small absolutely; all the energy that can be safely given a fused silica tuning-fork by a pianoforte hammer would not suffice to heat the fork 0.001°C.

The vibrations of the tuning-fork under consideration may be represented[2] by the well-known differential equation

$$\frac{d^2y}{dt^2}+k\frac{dy}{dt}+h^2y=0, \tag{1}$$

where k is the coefficient of damping and h^2 is the coefficient of stiffness.

The general solution of the equation is

$$y=Ae^{-kt/2}\cos\left(\tfrac{1}{2}(4h^2-k^2)^{1/2}\cdot t-\alpha\right), \tag{2}$$

which represents a simple harmonic vibration with decreasing amplitude, such as that produced on the photographic film, if k is positive.

The frequency of vibration n is given by the coefficient of t in the cosine term,

$$n=\tfrac{1}{2}(4h^2-k^2)^{1/2}; \tag{3}$$

and the period of a complete vibration T is

$$T=\frac{2\pi}{n}, \tag{4}$$

or

$$T=\frac{4\pi}{(4h^2-k^2)^{1/2}}. \tag{5}$$

The natural logarithm of the ratio of two successive maxima on the same side of the position of equilibrium is the logarithmic decrement

$$\delta=\log_e\frac{y_1}{y_2}=\frac{1}{2}kT \tag{6}$$

and it is a constant.

The rate at which the vibrations die down is proportional to k; and by putting equation (6) in the form

$$k=\frac{2\delta}{T}, \tag{7}$$

[2] A. G. Webster, Dynamics, pages 148–152.

it is obvious that the coefficient of damping of fused silica may be determined from a set of observations on the logarithmic decrement of this tuning-fork, for which T is known.

The coefficient of stiffness may be calculated from equation (5) when expressed in the form

$$h^2=\frac{16\pi^2+k^2T^2}{4T^2}. \tag{8}$$

The effect of damping on the period of vibration of the tuning-fork may be determined by assuming that there is no damping, i.e. $k=0$, in which case the period will be

$$T_0=\frac{2\pi}{h}.$$

When these values of T_0 and δ are introduced, the equation for the period becomes

$$T=T_0\left(1+\frac{\delta^2}{4\pi^2}\right)^{1/2}=T_0\left(1+\frac{\delta^2}{8\pi^2}+\cdots\right). \tag{9}$$

EXPERIMENTAL RESULTS

(a) Period of Vibration

In order that the values of the coefficients of damping and of stiffness of fused silica might be calculated from the data of Table 1, it

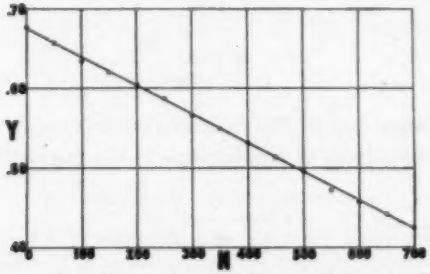

FIG. 1. *Number of vibrations versus logarithms of amplitudes.*

was first necessary to determine the period of vibration of the tuning-fork. The frequency was therefore observed by means of a monochord, and was found to be 274. The period, by equation (4), is then

$$T=\frac{2\pi}{274}=2.29\cdot10^{-2}\text{ seconds}.$$

(b) Logarithmic Decrement

The logarithmic decrement is calculated from values of the maximum elongation obtained from Fig. 1, plotted from data in Table 1, in which abscissas N represent the total number of vibrations after an arbitrary starting point and ordinates Y represent the natural logarithms of the corresponding amplitudes. Thus, on substituting in equation (6) the values of Y at the start and after seven hundred vibrations, the logarithmic decrement is

$$\delta=\frac{1}{700}\log_e\frac{y_0}{y_{700}}=\frac{1}{700}(\log_e y_0-\log_e y_{700})$$

$$=\frac{1}{700}(0.6762-0.4254)=3.6\cdot10^{-4}.$$

(c) Coefficient of Damping

The coefficient of damping of fused silica, as explained in connection with equation (7), may be calculated from a knowledge of the period and the logarithmic decrement of the tuning-fork. When the calculated values of T and δ are substituted in equation (7), k becomes

$$k=\frac{2\times3.6\cdot10^{-4}}{2.29\cdot10^{-2}}=3.14\cdot10^{-2}$$

(d) Coefficient of Stiffness

The coefficient of stiffness may also be computed from the values of T and δ by using equation (8). Thus

$$h^2=\frac{16\pi^2+(3.14\cdot10^{-2}+2.29\cdot10^{-2})}{4(2.29\cdot10^{-2})^2}=7.53\cdot10^4$$

(e) Effect of Damping on Period of Vibration

The influence of damping on the period of vibration is determined by substituting the value of δ in equation (9),

$$T=T_0\left(1+\frac{(3.6\cdot10^{-4})^2}{8\pi^2}\right)=T_0(1+1.6\cdot10^{-9}).$$

Thus k affects the period of the tuning-fork by less than two parts in a thousand million; so that, for a tuning-fork of fused silica, the period is essentially the same as if there were no dissipation. In other

words, the period depends on the structure of the tuning-fork, and not on the particular circumstances of the vibration.

CONCLUSIONS

Fused silica possesses many of the qualities which the ideal material for a tuning-fork should have: (1) its elasticity is high and its does not vary with time;[3] (2) the effect of temperature on its elasticity is small;[3,4] (3) its coefficient of damping, as this investigation shows, is low; (4) its "elastico-viscous flow"[5] is very small; (5) it has no "permanent set" or "lost motion,"[5] as is shown by the fact that fused quartz fibers used for suspensions are without drift at zero; (6) its coeffiicnet of expansion is extremely small;[6] (7) it is elastically isotropic,[3] as its coefficient of expansion is the same for weighted fibers and annealed rods.

It is highly desirable that the pitch of a tuning-fork shall not change with moderate changes of temperature. To obtain this result the tuning-fork should be made of such a substance that the effects of temperature on its elasticity and dimensions counteract one another. Whether fused silica has these properties may be determined as follows: The frequency of vibration n of a tuning-fork is proportional to

$$\frac{K}{L^2}\left(\frac{E}{\rho}\right)^{1/2},$$

where E is Young's Modulus, K the radius of gyration, L the length, and ρ the density. But with a given tuning-fork K varies as L, while ρ varies inversely as L^3. Hence

$$n \propto (E \cdot L)^{1/2}.$$

Now the coefficient of linear expansion[6] of fused silica at room temperature is about $+4 \cdot 10^{-7}$, while the temperature coefficient of elasticity[4] is about $+1 \cdot 10^{-4}$. The latter therefore, has by far the greater effect. But the condition for temperature compensation[7] for pure bending is that

$$\frac{1}{n}\frac{dn}{d\theta} = \frac{1}{2}\left(\frac{1}{E}\frac{dE}{d\theta} + \frac{1}{L}\frac{dL}{d\theta}\right),$$

[3] S. J. Barnett, Phy. Rev., *6*, p. 114; 1898.
[4] F. Horton, Phil. Trans., *204A*, p. 407; 1905.
[5] A. A. Michelson, Jour. Geol., July 1927 and January, 1920.
[6] W. Souder and P. Hidnert, Scientific Papers of the Bureau of Standards, No. 524.
[7] M. D. Hersey, Jour. Wash. Acad. Sci., 7, p. 437; Aug. 1927.

where θ is the temperature; and since both terms on the right side of the equation are positive, there can be no temperature compensation between elasticity and expansion in a fused silica tuning-fork.

The chief objection to the use of fused silica for a tuning-fork is its fragility. In order that Professor Webster's tuning-fork should not be broken it was necessary to strike it such light blows that the deflections at the ends of the prongs were seldom greater than 0.1 mm. The intensity of the sound emitted was therefore so small that the tuning-fork could not be used as a practical standard of pitch.

Another objection is that the vibrations of a fused silica tuning-fork cannot be maintained electrically, either by means of an electron tube or a mercury interrupter, without loading the prongs with iron; and then the vibrating system no longer consists entirely of fused silica.

A comparison of the results obtained for fused silica in this experiment with those for a steel tuning-fork is obviously impossible, because of the effects of size, shape, method of construction, ingredients, heat treatment, etc., in case of steel.

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A RECORDING PHOTOELECTRIC COLOR ANALYSER

By Arthur C. Hardy

INTRODUCTION

The instrument described in this paper was designed for the rapid measurement of the color of reflecting surfaces, a problem which is now of considerable importance in many branches of science and industry. It is quite generally agreed that, except for certain special purposes, a spectrophotometer or color analyser gives information of greater utility than is afforded by a colorimeter.[1] Also, the results obtained with the former are independent of the characteristics of the observer's eye and of the quality of the light source used for illuminating the sample. Spectrophotometers are notoriously tedious to use, however, and their precision is frequently insufficient, especially at the violet end of the spectrum. The present instrument was designed to eliminate these objectionable features.[2]

The operation of this instrument may be briefly described as follows: the specimen and a surface of magnesium carbonate are illuminated from opposite sides of a ribbon filament tungsten lamp. Beams of light from the specimen and from the magnesium carbonate standard are alternately admitted to the slit of a dispersing system. A narrow band of each of the spectra thus formed falls successively on a photoelectric cell and causes a pulsating current in the cell circuit when the two beams are of unequal intensity. This current is amplified by pliotrons and the alternating component is applied to the field coils of a small motor, the armature being supplied from an independent source of the same frequency. With this arrangement, the armature rotates in one direction when the beam from the specimen is more intense and in the opposite direction when the beam from the standard is more intense. When the intensities are equal, the field current ceases and the motor stops. By causing the motor to control a diaphragm which varies the illumination on the magnesium carbonate, the intensities of the two beams are automatically balanced. A pen operating on a rotating drum records the position of this diaphragm while the

[1] Ives, H. E., A Proposed Standard Method of Colorimetry, J.O.S.A. & R.S.I., *5*, p. 469; 1921.

[2] No attempt has been made to include a bibliography of the subject in the present paper. This has been rendered unnecessary by the excellent reports of the Progress Committee on Spectrophotometry published in this Journal. See J.O.S.A. & R.P.I., *10*, p. 169; 1925, and J.O.S.A. & R.P.I., *11*, p. 357; 1925.

rotation of the drum changes the wave length of the spectral band admitted to the photoelectric cell. In this way, a complete spectrophotoelectric curve for the entire visible spectrum is traced in approximately thirty seconds. The precision has been found more than adequate for all colorimetric purposes.

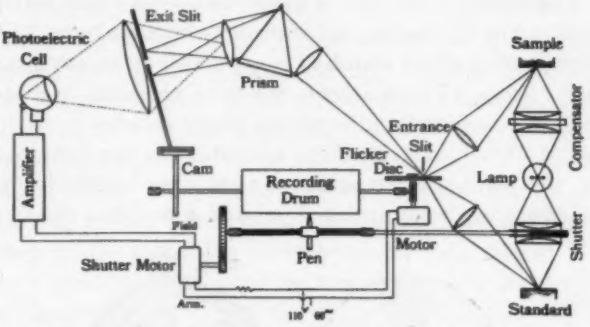

FIG. 1. *The optical system of the color analyser with a schematic representation of the recording mechanism.*

The special features of this instrument are: the method of illuminating the sample, which provides an extremely high level of illumination without injury from overheating; the elimination of the necessity for constancy of the photoelectric cell characteristics; and the fact that the instrument is recording and does not require an operator with specialized training in optics. In common with other physical spectrophotometers, this instrument has ample sensitivity at the violet end of the spectrum where visual spectrophotometers must always fail because of the low visibility of violet light.[3]

THE OPTICAL SYSTEM

The optical system of this instrument and a schematic representation of the recording device are illustrated in Fig. 1. The light source in use at present is a 5 volt, 45 ampere ribbon filament lamp operated

[3] The relative visibility of blue light as given by Gibson and Tyndall is only 0.043 at 440 millimicrons. Because of the difficulty of making accurate photometric settings when the visibility is low, it is quite common in the practice of visual spectrophotometry to ignore the violet region beyond 430 or 440 millimicrons. An attempt is sometimes made to justify this procedure on the grounds that light of low visibility is unimportant for colorimetric purposes. It should be remembered, however, that the color sensation evoked by a given specimen depends on the extent to which the three elementary color sensations are stimulated. The maximum of the violet sensation occurs at 440 millimicrons, and spectrophotometric data for colorimetric purposes should be determined to 400 millimicrons at least, and possibly beyond.

from two storage batteries of 150 ampere-hour capacity in parallel. One side of the ribbon filament is focused on the sample by means of a pair of Bausch and Lomb aspheric condensers of 60 mm focal length, having a numerical aperture of approximately 0.45. The other side of the filament is focused by means of a similar pair of lenses on a block of magnesium carbonate. A water cell between each pair of lenses (not shown in Fig. 1) absorbs part of the heat from the lamp.

The illumination of the standard is controlled by means of a shutter between the lenses. This shutter is of a type which has been described recently by L. A. Jones[4] and is shown diagrammatically on the right in Fig. 2. Although other methods of controlling the illumination are available, this method is non-selective as to wave length, is unaffected by zonal errors in the optical system, and provides a linear scale of

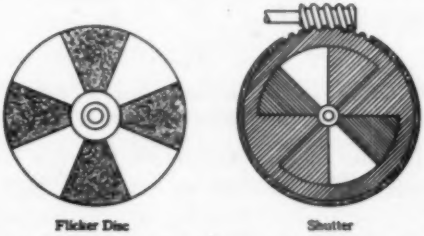

FIG. 2. *The flicker disk shown on the left consists of alternate silvered and clear sectors. The rotation of this disk causes the light reaching the photoelectric cell to come alternately from the standard and the sample. Changes in the sensitivity of the cell are of no consequence with this flicker method of comparison. The shutter shown on the right controls the illumination of the standard.*

reflecting power values. There is also a compensator of similar construction between the other pair of lenses, which will be discussed later.

The ribbon filament lamp is operated at a temperature of approximately 3000° Kelvin, the exact temperature being unimportant with this comparison method. The illuminated portions of the sample and the standard are rectangular areas 5 by 7 mm in size, although only a part of this area sends light to the photoelectric cell. The illumination with the shutter wide open is approximately 150 lumens per square centimeter, which is 15 times more intense than full sunlight on a clear summer day, and is many times greater than is available in any other spectrophotometer so far as we know. In spite of this high level of

[4] Jones, Lloyd A., An instrument (Densitometer) for the Measurement of High Photographic Densities. J.O.S.A. & R.S.I., 7, p. 231; 1923.

illumination, there is little difficulty from the heating of the sample. This is due in part to the small size of the illuminated area and in part to the water cell, which removes a good fraction of the heat radiation. Also, the air in the vicinity of the sample remains cool. In the case of very delicate specimens, such as the petals of certain flowers, the sample may be rotated as a further protection.[5]

The remainder of the optical system consists of a monochromator for analyzing the light and a flicker disk which admits light alternately from the specimen and from the magnesium carbonate. The flicker disk is of glass, 14 cm in diameter, and consists of four silvered sectors and four clear sectors as shown on the left in Fig. 2. It is rotated by a synchronous motor at a speed of 15 revolutions per second, giving 120 alternations of the two beams per second. The sample and the standard are focused on the slit of the monochromator by a pair of 16 diopter spectacle lenses, the illuminated area of the sample being more than sufficient to cover the width of the slit, although it fills only about 5 mm of its height as arranged at present.

As will be seen from Fig. 1, the optical system of the monochromator itself contains no new features. The wave length of the spectral band admitted to the photoelectric cell is selected by a movable exit slit. A large condenser behind the slit forms an image of the telescope lens on the cathode of the photoelectric cell, so that the cell itself may remain stationary.[6] The purity is maintained constant by means of a cam which automatically adjusts the width of the exit slit as the wave length is changed. In nearly all of the work so far the spectral band has been 10 millimicrons or less in width. Further improvements are contemplated which will result in greater purity in the regions of particular interest in colorimetry with an automatic decrease in purity toward the far red and far violet where the sensation values are low.[7]

[5] For certain purposes it is desirable to do this anyway to average the color of a complicated pattern which is intended to be viewed at a great distance. For example, certain types of roofing material consist of a mixture of crushed green slate and red brick dust pressed into an asphalt-coated felt base. On close inspection, both the red and green particles are visible. When the material is in place on a roof, however, these particles cannot be resolved and the surface appears homogeneous. Consequently, rotating the specimen gives the average color that is desired.

[6] This result could have been accomplished equally well with a constant deviation type of monochromator system.

[7] As a practical matter, an automatic adjustment which keeps the photoelectric cell current constant is not far from the best compromise between purity and sensitivity when a caesium cell is used. This has certain advantages from the standpoint of the design of the amplifier.

It may be remarked in passing that, with the present optical system, the total light flux received by the photoelectric cell depends principally on the illumination of the standard and sample and the size of the prism, the relative aperture of the monochromator system being unimportant. The illumination of the sample and standard is determined, of course, by the relative aperture of the condensers, but this is already about as high as it can be made conveniently. The lenses between the sample or standard and the slit need to fulfill only the following requirements: first, the images of the illuminated areas must slightly more than cover the slit; and, second, the angle subtended by these lenses from the slit must be at least as great as the angle subtended by the collimator lens. If these conditions are fulfilled, the slit appears as bright from the collimator lens as the sample or standard. The size of the prism determines the diameter of the collimator and telescope lenses, while the focal lengths may be chosen almost at will. If the focal lengths are too short the second requirement just mentioned is difficult to satisfy, but if they are too long the instrument merely occupies more space in the room, but there is no loss of light. With a given purity, the total flux received by the cell is proportional to the effective area of the prism and its dispersion. However, the amount to be gained by the use of a prism of very high dispersion is more than offset by the increase in absorption, especially at the violet end of the spectrum, because of the characteristics of the glass that must be used.

A complete specification of the color of a reflecting surface involves in general an infinite number of measurements, since the color of most substances depends on the directions of illumination and of viewing. Usually it is sufficient, however, to know the color for some standard set of conditions. One method in common use is to illuminate the specimen by a completely diffuse source, and to view it in a direction normal to the surface. It is possibly better to illuminate the specimen at an angle of 45° and to view it normally, since this corresponds more closely to the conditions under which colors are ordinarily seen or compared. In the present instrument this method was used, but the directions of illumination and of viewing were interchanged. By the principle of the reversibility of the light path, the results are the same as with 45° illumination and normal viewing.[8]

[8] It is obviously impossible to adhere strictly to the conditions of normal illumination and 45° viewing, since this would require an illuminating and viewing system of zero numerical aperture. Consequently, it is more accurate to say that the chief ray of the pencils illuminating the center of the sample and the magnesium carbonate strike the surfaces normally. Similarly,

THE PHOTOELECTRIC CELL

In our first experiments we employed a Case thalofide cell as the light sensitive element.[9] Although this cell is very sensitive to radiation of long wave lengths, it is not satisfactory in the blue and violet.[10] The sensitivity of the thalofide cell depends on its previous history and changes also with temperature, but this erratic behavior is of no consequence with the flicker method of comparison. The sensitivity must be sufficient, however, to produce a change in cell current capable of amplification; that is, the change in current due to the signal must exceed by a safe margin the accidental current variations in the cell.

The instrument is equipped at present with a caesium photoelectric cell (Type P.J.-15) manufactured by the General Electric Company. The spectral sensitivity of this cell extends sufficiently beyond the

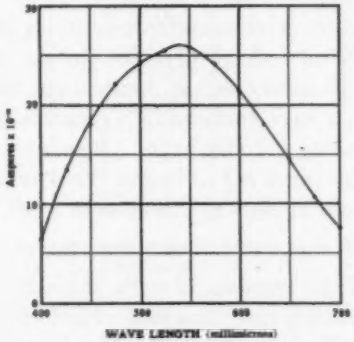

FIG. 3. *This curve shows the photoelectric current obtained at each wavelength with the present optical system from the magnesium carbonate standard with the shutter wide open.*

limits of the visible spectrum in both directions to fulfill the requirements of accurate color measurement. Also, the relatively uniform sensitivity of this cell throughout the visible spectrum makes the effect of stray light of much smaller consequence than in visual instruments where the variation in retinal sensitivity throughout the spectrum is much greater. Except for its bearing on the effect of stray light, the absolute value of the sensitivity of the photoelectric cell is not as useful

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the chief rays of the reflected pencils leave the surfaces at 45°. To specify the conditions completely, it is necessary to state also the numerical apertures of the incident and reflected pencils.

[9] Case, T. W., "Thalofide Cell—A New Photoelectric Substance," Phys. Rev., *15*, p. 289; 1920.

[10] This is true also of the improved cell which was later developed.

for the present purpose as a curve showing the actual cell current obtained in the instrument at each wave length (see Fig. 3). This curve was obtained with the shutter wide open and the magnesium carbonate standard in place. The entrance slit had a constant width of 0.7 mm while the width of the exit slit was automatically adjusted from 0.7 mm at 700 millimicrons to 6 mm at 400 millimicrons in such a manner that the width of the spectral band was always 10 millimicrons. More light could have been obtained in the short wave-length region by making the entrance slit adjustable also but this was not feasible in the present instrument. It is being done in the next model in order that the instrument may be made to operate farther into the ultraviolet. As the current at 400 millimicrons is only about 6×10^{-10} amperes with magnesium carbonate, the current produced by a black specimen is somewhat difficult to amplify.

One of the difficulties encountered in amplifying these small currents was traced to electrical leakage over the surface of the glass of the photoelectric cell. To reduce this to a minimum, the cell and the first stage of the amplifier were enclosed in a separate hermetically sealed compartment containing a drying agent. A guard ring (shown in Figs. 4 and 5) was likewise employed to reduce the potential difference over the glass from 150 volts or more to a fraction of a volt.

THE FIRST STAGE OF AMPLIFICATION

The first stage of the amplifier is really the heart of the instrument and was a source of annoyance for a long period. Since the cell current obtained at 400 millimicrons from a white sample is only 6×10^{-10} amperes, a change of 0.1% in reflecting power produces a change in cell current of only 6×10^{-13} amperes, and this must control the motor.[11]

The conventional method of coupling a photoelectric cell to a vacuum tube, and the one which we tried first, is shown on the left in Fig. 4. The cell is in a series circuit consisting of a battery, a one megohm resistance, and the resistance R. The one megohm resistance with the one microfarad condenser acts as a filter to allow the battery to be placed outside the shield. Variations in the cell current flow through the resistance R and cause corresponding changes in the voltage of the grid. The latter is maintained at the proper average potential by means

[11] It should be noted that a change of 0.1% in a white specimen produces the same change in cell current as a 1% change in a sample reflecting only 10%. However, by the present method of coupling the cell to the amplifier, the output depends on the fractional change rather than the absolute value. It is thus possible to measure all specimens to the same percentage accuracy.

of the blocking condenser C, a suitable grid battery, and a grid leak resistance r. The compartment containing the cell and first stage must be shielded acoustically, mechanically, and electrostatically. Any mechanical vibration of the tube elements or connecting wires appears as a stray voltage on the first grid which may easily exceed the signal in magnitude.

There are several difficulties with this method of amplification. To avoid ionizing the residual gas in the vacuum tube, the plate and shielding-grid voltages must be below the ionization potential.[12] We found it satisfactory to use the same 12 volt battery for both the shielding grid and the plate supply, and to choose the plate circuit

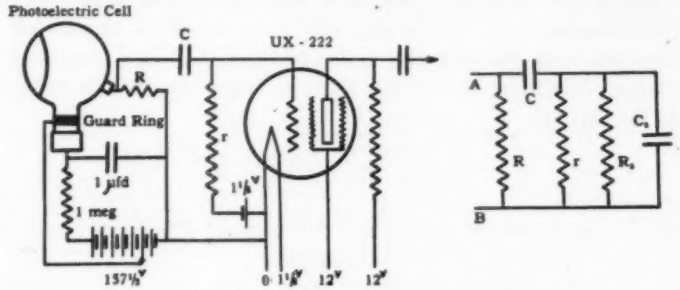

FIG. 4. *Diagram of the conventional method of connecting the photoelectric cell to the first stage. The equivalent circuit is shown on the right.*

resistance so that the plate voltage with normal plate current would be only six volts. Under these conditions, however, the first stage acts as an amplifier only when the grid voltage is maintained correct within a few millivolts. This is difficult to accomplish in the presence of an uncertain amount of leakage within the tube.

A further difficulty is that of obtaining suitable resistances for R and r. The proper value for these units depends on the frequency employed (in this case 60 cycles) and on the constants of the vacuum tube. The equivalent electrical circuit is shown on the right in Fig. 4, where C_g is the capacity to ground of the grid of the vacuum tube and associated elements and R_g is the resistance within the tube from grid to filament. When C_g is small compared with the capacity C of the blacking condenser, the latter may be neglected. There are then four parallel paths by which the alternating component of the photoelectric cell current may flow to ground. In the UX-222 tube, the grid connection

[12] General Ferrie, Quelques applications scientifique des lampes a 3 et 4 electrodes associees a des cellules photoelectriques. L'onde Electrique, *4*, p. 97; 1925.

is through the top and the grid-filament resistance is consequently much higher than in the average vacuum tube so that the path R_g can frequently be ignored.

Assuming that the cell current is large enough to charge the capacity C_g in a time which is short compared with half a cycle, the alternating component of the cell current I_c is a flat-topped wave representable by Eq. (1).

$$I_c=\frac{2}{\pi}\cdot\Delta I_c\left(\sin\omega t+\frac{1}{3}\sin 3\omega t+\frac{1}{5}\sin 5\omega t+\cdots\right) \tag{1}$$

In this equation, ΔI_c is the total change in cell current corresponding to a shift of the light from sample to standard or vice versa. As only the fundamental term is available for the operation of the motor, the useful voltage on the first grid is the product of $\frac{2}{\pi}\cdot\Delta I_c\cdot\sin\omega t$ and the impedance Z between A and B. The root-mean-square voltage is then given by Eq. (2):

$$e=\frac{2\cdot\Delta I_c}{\pi(2)^{1/2}}\cdot Z=0.45\cdot\Delta I_c\cdot Z \tag{2}$$

When C is large compared with C_g, the value of Z is given by Eq. (3).

$$Z=\frac{1}{\sqrt{\left(\frac{1}{R}+\frac{1}{r}+\frac{1}{R_g}\right)^2+\omega^2C_g^2}}\cdot \tag{3}$$

If R, r, and R_g are so large that the value of the parenthesis is negligible compared to $\omega^2C_g^2$, the root-mean-square voltage on the first grid by Eq. (2) is

$$e=0.45\frac{\Delta I_c}{\omega C_g}\cdot \tag{4}$$

Since the photoelectric cell current at 400 millimicrons with the present optical system is only 6×10^{-10} amperes, a 0.1 percent change in this current can produce at best an effective grid voltage

$$e=0.45\frac{6\times10^{-13}}{\omega C_g}=2.7\times10^{-13}\times\frac{1}{\omega C_g}\cdot \tag{5}$$

If the grid-filament capacity is 2×10^{-11} farads,[13] the signal voltage at 60 cycles under the conditions assumed is only 0.036 millivolts.

[13] The grid-filament capacity of the UX-222 tube alone is about 6×10^{-13} farads. The capacity of the entire circuit including the photoelectric cell is approximately 2×10^{-11} farads. With a screen-grid tube, the effective capacity is not increased by voltage amplification.

To obtain even this much signal, the resistances R and r theoretically must be infinite in comparison with the impedance of the capacity C_g. This requires resistances of the order of several hundred megohms or more, which are extremely difficult to obtain. Fortunately, the decrease in signal due to using lower values for R and r is not quite as serious as it appears at first glance. There are accidental variations in the grid current of the first tube which determine the smallest signal that can be amplified successfully. These current changes, which are a combination of shot and flicker effects,[14] appear as grid voltages when multiplied by the impedance of the path A B. Consequently, reducing the impedance by the substitution of lower resistances for R and r decreases the effect of the strays at the same rate as the signal. The

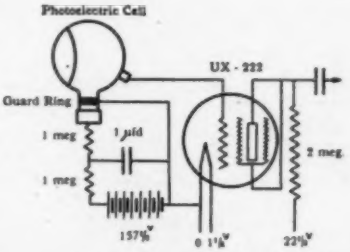

FIG. 5. *Diagram of a method of connecting the photoelectric cell to the first stage which allows a dark sample to be measured with the same accuracy as a light one. This method incidentally simplifies the design of the amplifier.*

smaller signal requires more subsequent amplification, but the ratio of the signal voltage to strays remains substantially constant.

A more satisfactory method of coupling the cell to the first stage, and the one which we are now using, is shown in Fig. 5. The anode of the photoelectric cell is connected through a suitable filter circuit to the positive end of the battery, the guard ring connection being used as before. The cathode is connected directly to the grid of the UX-222 tube used as a triode. In total darkness, the potential of the grid is substantially the same as with the photoelectric cell disconnected; that is, free-grid potential. When the cell is illuminated, the grid potential increases until the grid current is just equal to the photoelectric cell current. Fig. 6 shows the relationship of grid and plate

[14] Johnson, J. B., The Schottky-Effect in Low Frequency Circuits, Phys. Rev., *26*, p. 71; 1925.

currents to grid voltage for a typical tube of this type. The variation of grid current with grid voltage is expressed by Eq. (6):

$$\log_{10} I_g = 4.0\, E_g - 6.2 \tag{6}$$

Consequently,

$$dE_g = 0.25 \frac{dI_g}{I_g}. \tag{7}$$

Thus a change of 0.1% in I_g results in a signal of 0.25 millivolts on the grid of the first tube. This is independent of I_g so that the amplifier will work as well with a black as with a white sample.[15] This assumes,

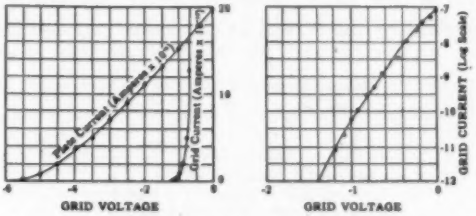

FIG. 6. *Curves showing the variation of plate and grid currents in the UX-222 tube used under the conditions shown in Fig. 5.*

of course that ΔI_g is large enough to charge the grid capacity in a time which is short compared to half a cycle. The quantity of electricity which flows into the grid during the charging interval is given approximately by Eq. (8).

$$Q = \Delta I_g \cdot t = C_g \cdot E_g \tag{8}$$

If we assume t=0.00833 seconds corresponding to 60 cycles, $C_g = 2\times10^{-11}$ farads, and $\Delta I_g = 6\times10^{-13}$ amperes, the value of E_g is 0.25 millivolts. A more accurate analysis would include the change in grid current also but this treatment is sufficient to show that the photoelectric cell current must not be less than is available at present if it is intended to work at 60 cycles. It should be noted [Eq. (7)] that the effect of stray light of constant intensity is to lower the precision. With the first method of connecting the cell to the amplifier, stray light of constant intensity has little effect.

THE AMPLIFIER

The voltage amplification in the first stage at present is approximately 2.5 times. Consequently a change of 0.1% in reflecting power produces

[15] The same result can be accomplished at the sacrifice of light by placing the shutter in the beam illuminating the sample rather than the standard.

a voltage change of 0.625 millivolts across the plate circuit resistance. As a 20 volt signal must be impressed on the grid of the power stage to start the motor, the total voltage amplification between first and last stages must be 32,000. The design of an amplifier capable of giving this much amplification with stability is not at all difficult at the present time. There is a special requirement to be met in this case which does not occur in the design of the average audio frequency amplifier. The current which flows through the photoelectric cell or the first vacuum tube is subject to accidental variations because of the random character of the electron emission. When the amount of available light is small, the shot or flicker effects[14] in the photoelectric cell or the first tube may produce strays of the same order of magnitude as the signal and thus overload the power stage. It is desirable, therefore, to tune the amplifier to the 60-cycle signal so that only the 60-cycle component of the shot or flicker effects will be amplified.

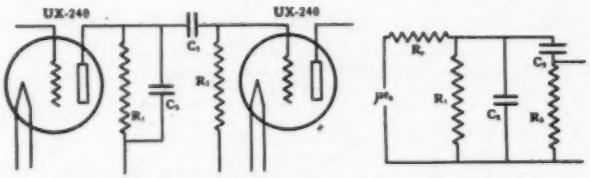

FIG. 7. *Interstage network of resistance-coupled amplifier used between the first stage and the power stage. By the choice of the proper values for C_2 and C_3, the amplifier is "tuned" to the frequency of the signal.*

Our early experiments were made with transformer coupling using a transformer with a peaked frequency characteristic, but a transformer in the early stages requires shielding against stray magnetic fields, and this can not be done very effectively at 60-cycles. We have, therefore, employed a circuit which may be crudely termed a "tuned resistance-coupled amplifier." The interstage network of this amplifier is shown at the left in Fig. 7. This is a conventional resistance-coupled amplifier to which has been added a bypass condenser C_2. The equivalent circuit is shown on the right, where R_p is the plate impedance of the tube. We employed tubes of the UX-240 type with a grid bias of -1.5 volts and a plate battery of 135 volts. Under these conditions the average plate resistance R_p is 100,000 ohms. In the present amplifier, R_1 is 250,000 ohms, R_3 is 2 megohms, C_3 is 0.01 microfarad, and C_2 is 0.005 microfarad. The voltage change of the second grid is proportional to the current through the branch R_3. By the application of

Kirchoff's laws, the voltage amplification per stage can be shown to be given by Eq. (9):

$$V.A. = \mu(a^2+b^2)^{-1/2} \tag{9}$$

where

$$a = 1+\frac{R_p}{R_1}+\frac{R_p}{R_3}+\frac{R_pC_2}{R_3C_3}$$

and

$$b = \omega R_pC_2-\frac{R_1+R_p}{\omega R_1R_3C_3}.$$

The voltage amplification is a maxium when

$$\omega^2 = \frac{R_1+R_p}{R_1R_3R_pC_2C_3}.$$

The amplification per stage has been computed as a function of frequency by means of Eq. (9), assuming the plate resistance to be 100,000 ohms and the amplification constant of the tube to be 30 (see Fig. 8). The *resonance* effect in a single stage is not very marked, but when

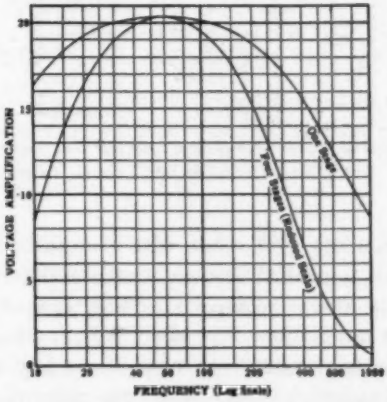

FIG. 8. *Curves showing computed performance of the amplifier shown in Fig. 7.*

four stages are cascaded, there is a definite maximum at 60 cycles and a decided falling off in amplification at higher and lower frequencies. This *tuning* is obtained with perfect stability and a high degree of immunity to stray electrostatic or electromagnetic fields.

THE METHOD OF RECORDING

It is obvious that readings with this instrument could be made by adjusting the shutter between the lamp and standard until the signal

reaches a minimum at each point in the spectrum as indicated by headphones in the output circuit of the amplifier. The fractional opening of the shutter would then be a measure of the reflecting power of the specimen at each wave length, but it is doubtful that this plan would have any merit over existing physical spectrophotometers.[16] By substituting a vacuum tube detector for the headphones in the output circuit, a relay may be actuated at the moment when the signal becomes less than any predetermined value. In the original scheme for this instrument, it was planned to have the illumination on the magnesium carbonate vary continuously, as by a continuous rotation of the shutter. The recording pen was to follow the movement of the shutter but was to make contact with the recording drum only at the moment when no signal was impressed on the detector. This was to be accomplished by means of an electromagnet operated by the relay in the plate circuit of the last stage. By connecting the drum to the mechanism for changing the wave length, a spectrophotometric curve consisting of a series of points could thus be obtained. Although recent trials have shown this method to be perfectly feasible, the present scheme of recording was developed in an effort to increase the speed of the instrument as much as possible.

Before the present plan was put into operation, several other methods offering the possibility of greater rapidity were devised and tried. In one of these, the amplified cell current was rectified by means of a commutator driven in synchronism with the flicker disk. In this case the direction of the rectified current depends on the relative intensities of the beams from the sample and standard. This current can be used to operate a relay of the double contact type which can start, stop, or reverse the shutter motor. The result produced in this way is a continuous curve rather than a series of points. This method has the disadvantage, however, that any extraneous disturbance in the amplifier which causes the latter to block momentarily usually holds the relay contact closed on one side and causes the motor to run at high speed away from the balance point, a difficulty which the present method avoids.

We had planned for some time to excite the field of the shutter motor directly from the output of the last stage of the amplifier and thus avoid troublesome relay contacts. This was not practicable, however,

[16] Gibson, K. S., Photoelectric Spectrophotometry by the Null Method, B. of S. Sci., Paper No. 349; 1919.

until power tubes were available. The field winding of the shutter motor requires about 10 watts of power when starting and has an impedance of about 400 ohms. By the use of two power tubes of the UX-250 type and the proper output transformer, it is not difficult today to match the tube impedance to the load and to supply the necessary power output. As tubes of this type were not available when this scheme was first tried, we used eight UX-171 tubes in parallel for the last stage with separate blocking condensers and leaks for each. The 220 volt dc mains were used for the plate supply with a 13 henry choke in the plate circuit to provide a return path for the plate current. One end of the field winding of the motor was connected to ground and the other to the plate through a 7 microfarad condenser. This capacity was sufficient to tune the motor field circuit to series resonance, thus reducing the impedance until it matched approximately the eight UX-171 tubes in parallel.

One of the principle advantages of driving the motor direct from the vacuum tubes is the extreme simplicity of the scheme. The only moving parts are operated by electric motors and there are no relay contacts to stick or get out of adjustment. Also, a temporary blocking of the amplifier[17] does not cause a lengthy excursion of the recording pen as it does when the motor is operated by a relay, since, with the present method of recording, blocking of the amplifier merely causes the motor to hesitate for a moment or to remain at rest if it was at rest when the blocking occurred. The time-constant of the grid circuit is $R_3 C_3$, so that a single stage of the present amplifier will clear itself in 0.02 second. Four stages will therefore unblock in 0.08 second.

The time required for the recording of a complete curve is determined principally by mechanical considerations. As the instrument is arranged at present, the shutter motor is geared down ten to one by a pair of spur gears, the result being that the effective moment of inertia is almost entirely in the motor armature. If, while the shutter motor is rotating at full speed in one direction, an obstruction is suddenly placed in the more intense beam, nearly half a second is required for the motor to come to rest. If a sample is suddenly inserted in the holder when the instrument is far out of balance, the motor runs beyond the balance point and makes several oscillations before coming to rest, the total time required in this case being about 3.5 seconds. Consequently, with the present apparatus, a complete curve representing

[17] Blocking of the amplifier is always the result of extraneous disturbances, such as a mechanical shock to the first tube, rather than too much signal.

50 points in the visible spectrum could be determined in 175 seconds with a precision of 0.1 percent or better at each point. When the wave-length slit moves continuously, the average time for a satisfactory curve may be somewhat less, and satisfactory curves for a gray sample have been recorded in as short a time as fourteen seconds. The average time, however, is about thirty seconds. By using slightly more power to drive the motor, a lower gear ratio and some damping in the mechanical system, we anticipate that the time may be still further reduced.

Theoretically, there is always the possibility of encountering a sample having a perfectly sharp absorption band which the instrument would not follow. This difficulty is more academic than real, but

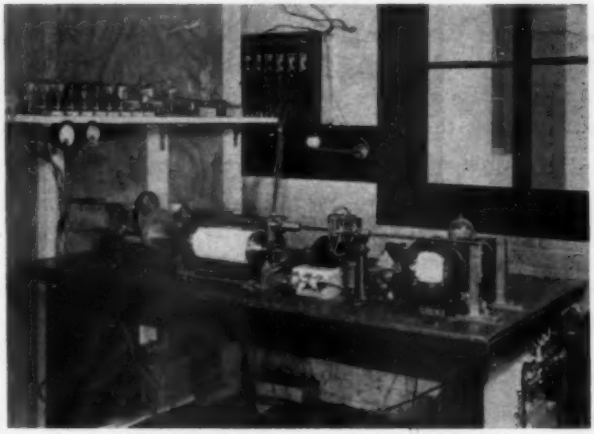

FIG. 9. *Photograph of first model of color analyser.*

may be solved by the use of a step wave-length cam. We have tried also various speed regulating devices which stop the drum when the instrument is out of balance by more than a predetermined amount. One simple method is to connect a vacuum tube detector to the last stage. By the use of a circuit-breaking relay in the plate circuit of the detector, the drum motor operates at full speed when measuring a specimen with a relatively flat spectrophotometric curve and automatically slows down if the curve becomes steep. However, the time required at present is so short that these devices are probably not necessary. We did find it advisable to include a safety switch at each end of the drum to open the motor circuit and prevent the jamming

of the apparatus by the accidental removal of the sample or the obstruction of one of the beams.

Fig. 9 is a photograph of the first model of this apparatus. The lamp and magnesium carbonate block are conspicuous at the right, while the flicker disk and entrance slit of the monochromator lie slightly to the left of the magnesium carbonate block. The photoelectric cell and the first stages of the amplifier are inside the cage at the left. The recording drum is near the center of the table. It accommodates a sheet of paper $8\frac{1}{2} \times 11$ inches in size[18] which can be perforated to fit the pins on the periphery of the drum. We intend soon to have paper especially printed with suitable coordinates.

PRECISION

The precision of this instrument can be stated most conveniently by considering separately its reproducibility and its accuracy. The former is an indication of the freedom from accidental variations of its indications while the latter is largely determined by the care used

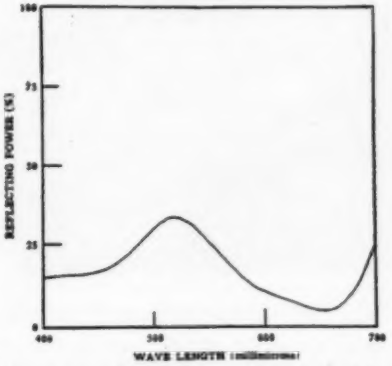

FIG. 10. *A typical curve representing in this case the analysis of the color of a piece of green silk. This specimen was measured twice by the instrument and the differences between the two curves are less than the width of the pen line.*

in construction and calibration of the apparatus. When the wave length of the light is constant, the reproducibility of reflecting power readings is 0.05 percent or better.[19] When the drum is in motion, the

[18] A sheet of this size affords ample precision for reading the record except for specimens of very low reflecting power. In the latter case, it is often convenient to place a circular stop in the beam of light illuminating the standard, thereby expanding the scale of reflecting power values.

[19] There is theoretically no lower limit to the precision of the flicker method. The practical limit depends on the ratio of the signal to the strays.

inertia of the recording mechanism causes a lag which may, in the case of a steep curve, introduce an error as great as 0.5 percent. In general, however, the present model, crude as it is, will retrace the same curve many times with a departure less than the width of the pen line. Fig. 10 shows a typical curve for a specimen of green silk which was measured twice by the instrument. It will be seen that the accidental variations in performance were not serious enough to widen the pen line appreciably. In fact, several thousand curves have been made on this instrument during the last year and its reproducibility seems to be entirely satisfactory for colorimetric purposes.

As the present model was constructed for experimental purposes, no great care was taken to make it a precision instrument, although some of the common causes of inaccuracy were eliminated at the start. For example, the optical system from the entrance slit of the monochromator to the photoelectric cell is simpler than that of visual spectrophotometers and gives no appreciable error from stray light.[20] Since exactly the same area of the photoelectric cell is illuminated by the sample and the standard, there is no difficulty from local variations in sensitivity over the surface of the cathode. Slow changes in the cell sensitivity are rendered immaterial by use of the flicker principle. The path of light from the lamp to the entrance slit of the monochromator via the sample is similar to the path via the standard with the exception of a reflecting surface on the back of the flicker disk. As silver is selective in its reflecting power, a compensator is inserted between the lamp and the standard. This is a shutter operated from a cam on the drum shaft, the cam being so fashioned that a specimen of magnesium carbonate reads 100 percent at all wave lengths. Any difference in the emissivity of the two sides of the filament is compensated in this way as well as the selective reflection of the silver.

The accuracy of the instrument was tested by measuring a filter lent by Mr. I. G. Priest of the Bureau of Standards. A block of magnesium carbonate was placed in the sample holder and the filter was mounted between this and the flicker disk. Since the optical parts in the present model are uncorrected spectacle and condenser lenses and no especially accurate results had been expected, the results of

[20] This system conforms closely to the principles enumerated by H. E. Ives in his paper on "Scattered Light in Spectrophotometry and a New Form of Spectrophotometer," Phys. Rev., *30*, p. 446; April 1910. When in proper adjustment, the error due to stray light is everywhere less than 0.1% as evidenced by the failure of filters inserted just before the cell to cause any perceptible change in the balance point.

the test, shown in Fig. 11, were extremely gratifying. The upper curve is for the magnesium carbonate with the filter removed. The compensator had not been connected when this test was made so the values furnished by the Bureau of Standards had to be multiplied by the ordinates of this curve to be comparable with the values plotted by the instrument. The deviation of the lower curve from the plotted points

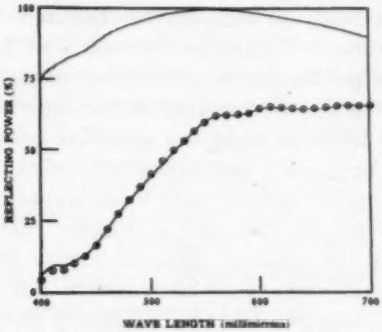

FIG. 11. *Results of a test of accuracy using a piece of amber glass as a specimen. The lower curve was determined by the present instrument while the plotted points represent data furnished by the Bureau of Standards. The cause of the error at 410 millimicrons has been discovered and eliminated.*

is thus a measure of the accuracy of the present instrument. There is an error in the region around 410 millimicrons which was later traced to stray light leaking around a diaphragm. Unfortunately, the sample had been returned before this was discovered and remedied, but we feel confident that with the instrument adjusted as at present, any reflecting or transmitting specimen can be measured with an accuracy sufficient for all practical colorimetric purposes.

FURTHER IMPROVEMENTS

As is usually the case, the first model possesses many minor defects which will be corrected in the one which we are now constructing. The new model will, in addition, include one change of major importance; namely, the lamp and the photoelectric cell will be transposed. This may be done without altering the total light flux received by the cell, but the sample will then be illuminated by monochromatic light instead of white light. This arrangement was considered before the present model was constructed but was not adopted because of the difficulties that arise in interpreting results with a fluorescent

specimen. On certain theoretical grounds it seems desirable that the instrument should measure the color of the specimen as illuminated by white light, including the fluorescence if present. By illuminating the sample with monochromatic light, the need for water cells vanishes. This feature more than outweighs the theoretical advantage of white light illumination because of the rarity of strongly fluorescent samples.

In the present model, any difference in emissivity between the two sides of the filament is corrected by the compensator. For precise results, a new cam must be cut whenever a new lamp is inserted. Consequently, the next model will have a slightly different optical system which will eliminate the mirror, but will cause the beams from the sample and standard to illuminate precisely the same area of the photoelectric cell as before. Several schemes for accomplishing this result are being tried out now.

THE INTEGRATING ATTACHMENT

One advantage possessed by a colorimeter which is not shared by a color analyser is the simple manner in which the results are expressed. Thus, the colorimeter describes by three simple numbers the color sensation produced by the specimen under a given set of conditions, whereas the color analyser requires a curve or a series of thirty or more numbers. Any two specimens representable by the same three numbers will appear to match under the same conditions of illumination if the observer has normal color vision, although the spectrophotometric curves in the two cases might be quite different. Of course, in the latter case, the two specimens would not match if illuminated by light of a different quality. Since some experience is necessary before a color sensation can be visualized even approximately from the spectrophotometric curve of the specimen, it is highly desirable to have a simple means for computing the sensation, if for no other reason than to make it possible to file together all curves for specimens which look alike under the assumed set of conditions.

A color sensation is described most conveniently by giving the degree of excitation of the three primary receptor mechanisms. To compute these primary excitation values from the spectrophotometric curve of the specimen, we assume first the energy distribution curve of the source to be used in viewing the specimen, for example, mean noon sunlight. This curve is then multiplied by the spectrophotometric curve of the specimen, ordinate by ordinate, to produce the luminosity curve of the specimen for the assumed source of illumination. The

ordinates of the luminosity curve are then multiplied by each of the three primary excitation curves.[21] The result of this operation is a set of three curves, the areas under which are measures of the three primary excitation values.

When performed in the ordinary manner, this process requires an hour or more for each specimen and is impracticable as a routine procedure. We are now constructing an integrating device to be combined with the analyser for computing mechanically the sensation values while the spectrophotometric curve is being traced. A number of well-known mechanical integrators have been described in the literature,[22] but the simplest and most adaptable for this work is the one shown in Fig. 12. The shaft S is connected to the recording drum

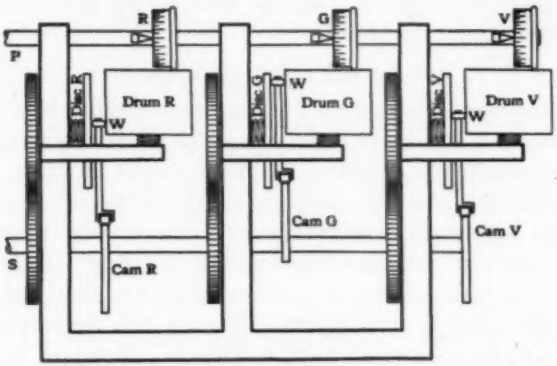

FIG. 12. *An integrating attachment for the color analyser which mechanically determines the three primary sensation values.*

in such a manner that it makes one complete revolution during the time that a spectrophotometric curve is being made. This shaft carries three cams R, G, and V, the radii of each of which are proportional to the product of the assumed energy distribution of the source and the three primary excitation functions. The disks R, G, and V rotate at constant speed. The three drums R, G, and V are rotated at a speed proportional to the radii of the respective cams by means of the wheels W in contact with the disks R, G, and V. The rod P carries three index wheels R, G, and V which are driven by contact with the ends of the three corresponding drums. One end of the rod is connected to the

[21] See Report of Committee on Colorimetry for 1920–21, J.O.S.A. & R.S.I., *6*, p. 548; 1922.

[22] V. Bush and H. L. Hazen, "Integraph Solutions of Differential Equations," Frank. Inst. J., *204*, p. 575; 1927.

recording pen so that the relative speed of each index wheel is proportional to the product of the speed of the corresponding drum and the instantaneous value of the reflecting power of the specimen.

It will be seen that this mechanism gives the integral of the product of each of the primary excitation functions by the energy distribution of the source and the reflecting power curve of the specimen. For convenience the cams can be so fashioned that a specimen of magnesium carbonate will read 100-100-100 or some other convenient set of numbers. There are other applications for an integrating mechanism of this sort. For example, a cam may be cut to correspond to the visibility curve, in which case, the dial indicates the visual brightness of the specimen. Other applications will doubtless suggest themselves to the reader.

In closing, the author wishes to express his indebtedness to Mr. Fred H. Perrin for his valuable assistance during the development of this instrument, especially during the early experiments which tested the basic principles on which the instrument operates. He wishes also to acknowledge the help of Mr. F. W. Cunningham, who later offered many valuable suggestions in connection with the use of thermionic tubes for the amplification of photoelectric cell currents. Mr. O. W. Pineo, Mr. David Luck and Mr. J. M. Nichols have made more recent contributions to the project.

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MOTION PICTURES IN RELIEF

By Herbert E. Ives

One of the outstanding challenges in the motion picture art is some method of projecting moving pictures so that they may be seen in stereoscopic relief. Several methods have been proposed, all of which involve the use of some special viewing device at the observer's face. Among these may be mentioned the anaglyph or two color arrangement which demands a pair of special colored glass spectacles; the polarization method in which the two images are polarized in different planes, and are viewed through analyzing prisms, one over each eye; and the method demonstrated several years ago under the name of "Teleview" in which the two images are projected alternately and are viewed through a sector disc which exposes each eye in turn. The dream of experimenters in motion picture projection has been some method by which relief could be obtained without demanding the use of any instrument at the observers' eyes.

It is the purpose of this note to describe a method in which, by utilizing the principles of the parallax panoramagram, this ideal of stereoscopic motion picture projection can, theoretically at least, be attained. The method, as will be seen, is relatively complicated and has severe practical limitations, but it appears to be theoretically sound and capable of realization at least on an experimental scale.

The kind of picture which I have called a "parallax panoramagram" consists essentially of a very large number of adjacent linear picture elements, each one of which contains a complete panorama of the scene presented to the camera.[1] The picture is made up in the form of a transparency and is placed a short distance behind an opaque line screen ruled so that the opaque spaces are very much wider than the transparent. When the transparency with its associated grating is viewed from any position, a composite image is seen, built up out of the narrow strips which are visible through the grating spaces. This view changes with the position of the observer, and is different for his two eyes so that from every position in front of the grating a stereoscopic image is seen.

It is immediately obvious that if a proper image of narrow parallel panoramic elements could be projected on a translucent screen in front of which was a grating of the sort described, the parallax pano-

[1] C. W. Kanolt, U. S. Patent, 1,260,682; 1918.

ramagram scheme could be carried out by projection. If furthermore, the projected image consisted of a series of pictures, following each other in rapid succession, motion pictures in relief could be achieved.

The whole problem appears to be that of producing the special image consisting of the multitude of linear panoramic elements. In the camera for producing parallax panoramagrams which I have recently described,[2] the negatives are produced by a continuous motion of the photographic lens, in a horizontal plane. The sensitive plate is placed a short distance behind the grating, and plate and grating move

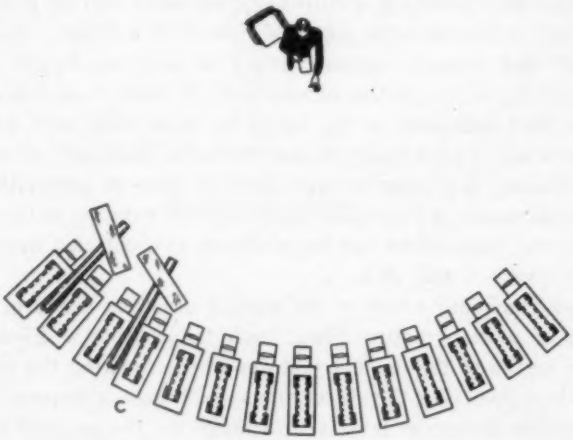


FIG. 1

both with respect to the object and to the photographic lens in such a manner that the desired panorama is developed behind each grating slit. While theoretically this method of making the panoramagram negative is applicable to making a series of views for motion picture purposes, the practical difficulty of moving the whole motion picture camera from side to side with the requisite speed or of attaining the equivalent by any reasonable optical means appears for the present insuperable. I have therefore been led to consider an alternative scheme as follows: Instead of a single lens moved from side to side forming the multi-line image on a single film, let us imagine a large number of motion picture cameras set up side by side, all directed to the same object as shown in Fig. 1. If these motion picture cameras can be made sufficiently narrow so that the lenses are in practical juxtaposition, we

[2] Ives, "A Camera for Making Parallax Panoramagrams." J.O.S.A. & R.S.I.; Dec. 1928.

can secure, by photographs on the entire number of films, a complete series of views of the object from all necessary directions. The number of cameras required, in order that there shall be no gaps in the photographic record, is of course very large; of the order of magnitude of 50 or more.[3] The number can, however, be kept within practical limits by utilizing the device of high speed motion picture photography. If instead of a single picture corresponding to each frame of the motion picture film as run at normal speed, several be taken, let us say 4 or 5, and if the point of view of all of the lenses be changed for each view, we can secure (assuming a corresponding rapid rate of projection), a practically infinite series of points of view. The necessary change of the point of view of each exposure might be achieved by an oscillatory motion of the whole system of cameras. A more elegant device, however, is that indicated in the figure in connection with four of the cameras where a thick plate of glass mounted diagonally on a rotatable axis is shown. If a series of such plates of glass of proper thickness is kept continuously or preferably intermittently rotating as the exposures proceed, the same effect can be produced as oscillating motion of the camera system would give.

We now have our record of the moving object in the form of a large number of motion picture films; each taken from a different angle and, by virtue of the devices just described, covering the entire wide angle whose apex is at the object, in as small angular steps as is desired. The problem before us now is to arrange for the projection of these film records in such a way as to secure a relief image. In Fig. 2 is shown one method by which this might be accomplished. This method consists substantially in reversing the optical paths; of setting up in place of the original cameras a set of motion picture projectors furnished with the rotating thick glass plates used in the negative making operation. The images from all the projectors are directed upon the translucent screen S. A short distance behind the screen, between it and the projectors, is placed an opaque line screen of the parallax panoramagram type with wide opaque and narrow transparent spaces. By virtue of this screen, the picture from each projector is cut up into narrow adjacent strips upon the screen. Imagine now an observer on the side of the transmission screen opposite to that of the projectors. The image on the screen appears to him exactly like the

[3] The practical juxtaposition of their points of view might be assisted by placing a second and third series of cameras pointing upward and downward, receiving their light from semi-transparent mirrors at 45° placed in front of the lenses of the first series.

parallax panoramagram positive before the opaque line grating is put in place. In order now for the picture to be seen in relief, it is only necessary to place in position a second grating as shown at G_2. When this is done, an observer at any position in front of this screen sees a motion picture in relief.

While the use of a large number of cameras for taking the original film negatives is a cumbersome and expensive arrangement, it is not

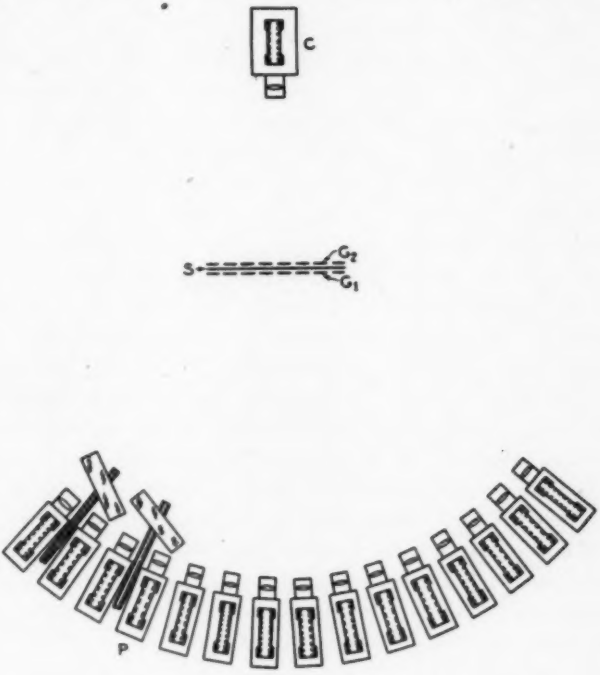


FIG. 2.

prohibitive in the studio. On the other hand, the use of a large number of projectors every time the pictures are to be viewed must be recognized as a very serious handicap. It is accordingly desirable if possible to greatly simplify the projecting scheme. This may be done by the following method: Suppose that instead of viewing the screen S through a parallax panoramagram grating, we set up a motion picture camera at some point such as C, and photograph the screen, receiving upon each frame of the film in the camera C, the composite picture formed

by the several pictures taken at slightly different angles in the original photographic operation. Or what amounts to the same thing, suppose that we use for the projectors in Fig. 2, a set of long focussed projectors each of which projects an image of ordinary film frame size upon S, which is now an ordinary motion picture film moved in the usual manner at an appropriate speed. By either of these procedures, we secure a film similar in its structure to the positive used in the parallax panoramagram.

The film which we have just obtained may now be projected from an ordinary projector upon a translucent screen. In front of the translucent screen, we must then place a parallax panoramagram grating. Provided the lines of the grating and the panoramagram are in proper registration with each other, we have the desired stereoscopic projection. The accurate registration of the projected image with the grating lines is the chief practical difficulty in this method but there is no reason to believe that it could not be satisfactorily solved.[4]

As has been pointed out in a previous paper in connection with the parallax panoragram, relief pictures of this sort suffer from certain limitations. The form of opaque line screen which is necessary is very wasteful of light, so that projection by any form of light except the most powerful arc lamp or to large sizes would be difficult. There is also the limitation that as long as the transparent spaces of the grating are kept of finite size, the definition of objects much removed from the plane of the screen is poor.[5] It is hardly to be hoped that with these limitations and with the complexity of apparatus which is required this method of stereoscopic motion picture projection can be expected to be entirely satisfactory. However, it is believed to be theoretically sound and to be the first method which does actually meet the ideal stated at the beginning of this article, namely a method of motion picture projection in relief which requires no apparatus at the observers' eyes.

BELL TELEPHONE LABORATORIES,
NEW YORK CITY, NOV. 12, 1928.

[4] Small, slow deviations from proper registration would have the effect of apparently slowly shifting the observer's position—such a shift introduced by moving the camera has been used purposely to give the illusion of relief in motion pictures.

[5] Both of these defects of the parallax panoramagrams can be materially lessened if, instead of gratings consisting of opaque lines, a structure of parallel cylindrical lenses is used. This form of grating would be entirely feasible in the relatively large screen involved in motion picture projection.

LENARD RAY TUBE WITH GLASS WINDOW

By C. M. Slack

In Lenard's[1] original tube electrons were projected through thin aluminum foil fastened to a metal support with wax. These Lenard rays were of low velocity and range due to the voltage limitation imposed by his form of tube. Since then there have been great improvements, the most notable of which were the introduction of the hot cathode by Eisenhut[2] and the employment of high voltage by Coolidge.[3]

The difficulties encountered in the construction of such tubes arise from the necessity of fastening a foil of the order of .001 cm in thickness to a metal support, sealing this to the glass envelope of the tube and then freeing these various parts from the occluded gas. These operations are costly and laborious and it is extremely difficult to operate such tubes off of the pumps without the use of charcoal and liquid air.

THIN GLASS WINDOWS

These difficulties have been largely overcome by the employment of thin glass windows. These windows are constructed by drawing in the end of glass tubing to form a bubble, the glass having been previously thinned at this point by working. Windows of this shape have been constructed as large as 6 cm in diameter. The size, of course, determines the thickness necessary to withstand atmospheric pressure. For instance, a window 2.5 cm in diameter and as thin as .0005 cm will stand atmospheric pressure when applied so as to put a tension strain in the glass. During the process of tube construction such windows must be protected by a pressure on the concave side as the slightest force from the glass-blower on the convex side would otherwise destroy them. However, for practical purposes, windows thick enough to enable them to be sealed in are satisfactory.

Though the melting point of pyrex glass is considerably less than that of nickel or molybdenum yet (since glass does not oxidize nor in any way deteriorate at temperatures below the softening point) it is probable that the windows on this tube can be run at temperatures comparable with those of the metals which have so far been used.

[1] P. Lenard, Ann. d. Phys., *51*, pp. 225–267; 1894.
[2] O. Eisenhut, Heidelb, Diss., 1921.
[3] W. D. Coolidge, Journal Franklin Institute, *202*, pp. 693–736; 1926.

We have baked these tubes to 500° C without any noticeable weakening of the window.

The formula[4]

$$V_0^2 - V_x^2 = K\rho X \text{ where}$$

V_0 = voltage through which electrons have fallen
V_x = remaining voltage
K = constant.
ρ = density of material in window
X = thickness of window

shows that the voltage or energy loss in the window is a function of

1. the density of window material
2. thickness of window.

It is thus desirable to keep these two factors as low as possible.

Of the metals, beryllium or aluminum would be desirable from the first of these considerations, but due to the fact that they cannot be soldered they are not suitable except as a laboratory experiment. Even on the pumps the wax used to seal the foils to their support prevents the employment of high voltages. Glass has a density close to that of aluminum and it is possible to construct suitable glass windows thinner than those made of metal.

There is no grid structure in this form of tube to intercept part of the rays. Due to these considerations it should be possible to send more energy per unit area through the glass than through the metals, such as nickel, copper or molybdenum.

The heat conductivity of the metal foils would aid in their cooling but under most conditions of operation the heat loss by convection plays the major role.

ENERGY LOSS IN WINDOW

Another interesting fact which follows directly from the formula is the reduction in energy or voltage loss in the window with increasing potentials across the tube. For instance, using a glass window .0005 cm in thickness and applying a potential of 20 kv across the tube practically all of the electron energy will be absorbed in the window whereas with 100 kv across the tube only 3% of the electron energy will be absorbed by the window and with 200 kv only 3/4 percent. Thus, contrary to the experience with x-ray tubes the electron current through the tube may be increased with the voltage without overloading the tube.

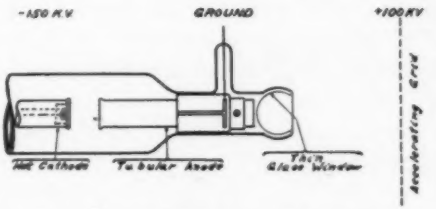
[4] Terrill, Phys. Rev., *22*, 101–108; 1923.

The anticipated difficulty of electrostatic discharges cracking the window was not encountered. It was found at first, however, that the lowest voltage at which electrons could be made to emerge from the window was a function of the size of the window and in every case greater than given by the preceding formula. Also, it was noticed that the tube operated better when the anode or window end was grounded. This was soon found to be due to a negative charge accumulating on the glass and air in the immediate neighborhood and acting very much like a negatively charged grid in a three-element vacuum tube. No current could pass until the voltage was raised sufficiently. This difficulty was completely overcome by putting a conducting coating on the outside of the glass from the anode lead-in and extending to a point beyond where the rays emerge from the window. The critical potential was thereby lowered from around 80 kv to 25 kv.

INCREASING VELOCITY OF EXTERNAL RAYS

This space-charge effect suggested the possibility of influencing the range and even the velocity of the cathode rays with potentials outside the generating tube.

Though it is impossible with any voltage to obtain electrons of appreciable velocity from rest in air since the potential gradient would have to be more than 100,000 volts per cm (far above the arcing potential), yet, given an initial velocity corresponding to the fall through



Lenard ray tube with external accelerating grid.

150 kv, the potential gradient in the air necessary to maintain this velocity drops from 100 kv per cm to 2 or 3 kv per cm. Thus, by the application of a potential gradient of a few thousand volts per cm outside the tube it should be possible to increase the range and available energy of the cathode stream indefinitely.

By making the cathode end 100 kv negative, grounding the anode and placing a metallic plate at 100 kv positive in front of the window outside the tube, we have shown this to be true in a qualitative way.

(See Fig. 1) With such an arrangement the range of the rays is materially increased. Due to the accumulation of negative space charge in the air between the external plate and anode, it is probable that at no time is the actual velocity greater outside the tube than inside; but an arrangement of the external potential in such a manner as to form a series of positively charge grids of increasing potential would remove this space charge and the restrictions which it places upon increase in velocity. However, even with an arrangement similar to that shown it is possible materially to increase the energy dissipated by the cathode ray stream.

GLASS WINDOW OSCILLOGRAPH

At present in our laboratories we are attempting to adapt this form of window to oscillographic uses and the results so far seem promising. At present there is no form of Lenard ray oscillograph. By this is meant an oscillograph in which the electron beam leaves a trace directly upon a photographic film placed external to the generating tube. We have mounted windows 5 cm in diameter and .001 cm in thickness in a tube and have been able to obtain a trace of simple transient phenomena such as the breaking of an electric circuit. It is necessary that the photographic plate be in contact with window to prevent scattering of the cathode ray beam by the air. For this purpose we use thin glass plates which have the same curvature as the window.

RESEARCH LABORATORY,
WESTINGHOUSE LAMP CO.,
BLOOMFIELD, N. J., NOV. 23, 1928.

A DESIGN OF A LOW VOLTAGE X-RAY TUBE

By Elmer Dershem

In any experimental study of long wave-length x-rays the research worker must turn to some type of x-ray tube which permits ready access to the electrodes for the purpose of cleaning or interchanging targets and replacing filaments if a hot cathode is used. There is little need that such a tube be designed to operate at potentials above 25 kilovolts but it is essential that it be capable of carrying the greatest possible current in order that a large amount of energy be available at low potentials. It is often much to be desired that it should operate satisfactorily at a vacuum obtainable without costly pumping equipment or the continual expense of liquid air.

For these reasons it seems worth while to describe briefly a design of tube[1] which has been in almost continuous operation during the past eighteen months and which seems in a large measure to meet the above requirements.

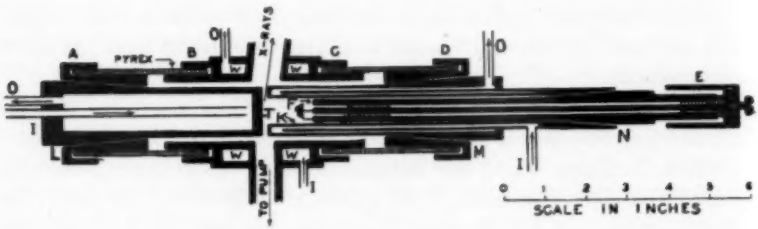
One of the well known phenomena of electrical discharge through rarefied gases is that a much greater potential is required to produce a spark discharge when the distance between electrodes is somewhat less than a mean free path of the atoms or molecules of the gas at the given pressure, than is the case if the electrodes are spaced at a greater distance. To understand this we must consider the fact that the current, due only to ions formed spontaneously in a gas, is vanishingly small. It is only when these ions are multiplied by collision with neutral atoms or molecules that electric breakdown occurs. The chance of an ion hitting a neutral atom or molecule may be made very small by reducing the length of path it may travel before reaching an electrode and giving up its charge.

These considerations indicated that by choosing the proper spacing between the parts having different potentials, an x-ray tube might be constructed which would exhibit no ionization current even with gas pressures much higher than are commonly thought permissible. Therefore experiments were undertaken to determine this optimum spacing for air pressures such as might easily be obtained with an oil pump. For air at pressures ranging from 0.002 to 0.01 mm Hg, this spacing

[1] Preliminary report presented at Nashville meeting of the American Physical Society. See abstract, Phys. Rev., *31*, p. 314; 1928.

was found to be approximately 1/8 inch. If the pressure was 0.01 mm Hg the maximum potential before the passage of a spark was about 20 kilovolts while with lower pressures much higher potentials were required to produce this result. This applies to plane surface electrodes which are not outgassed by long evacuation accompanied by heating or electrical bombardment.

If the gas is almost completely removed from an x-ray tube, as is the case with the commercial Coolidge tubes, the negative space charge (due to the current being entirely carried by the electrons) places a serious limitation upon the current capacity at low voltages. On the other hand, with higher air pressures the slow moving positive ions neutralize, to a large extent, the space charge due to the electrons. Bringing the electrodes close together also reduces the space charge effect. About the only drawback to the use of such high pressures is



LOW VOLTAGE X-RAY TUBE

the more rapid oxidation of the tungsten filament but insomuch as a filament will endure for 20 or more exposures of one hour each and may readily be replaced this is not serious. If air is not admitted to the tube between working periods the filament will last much longer. No experiments have been undertaken to determine the behavior of oxide coated filaments under these conditions but it is hoped that this may soon be done.

A section through the center of this x-ray tube is shown in the accompanying figure. The target T, cathode focusing tube K, and filament holder F, are each easily removable by means of the tapered ground joints at L, M, and N respectively. Cooling water enters through the tubes marked I and leaves through those marked O. The spaces marked W represent a water jacket in the central section of the x-ray tube. This water jacket encircles the part where heat is generated and prevents softening of the wax at B and C. The parts

shown in black are of brass except the portions nearest the hot filament which are of steel because of its higher melting point and better lasting qualities. Many of the parts represented in the drawing are, of course, made of separate parts soldered together. To show all details would necessitate the use of a much larger figure and this is not necessary, since any mechanic can work out suitable ways of assembling the parts which when soldered together, form one of the parts shown in the drawing.

A steel rod, insulated with sections of pyrex tubing, carries the current to the filament which is attached to the rod by means of a set screw. The other end of the filament is attached in like manner to the steel tube forming the outer part of the filament holder. The rod, at least, should be of steel rather than copper as in the latter case a rod of suitable stiffness would conduct sufficient heat from the filament to cause softening of the wax at *E*. The lengths of the rod and tube are made adjustable by means of telescoping sections (not shown in the figure, to avoid unnecessary complexity). The focal spot on the target may be made large or small by placing the filament at different distances from the end of the focusing tube. The spot may be made as fine as the point of a needle if desired.

The insulation between the electrodes and the central section of the x-ray tube is provided by two pices of pyrex glass tubing 3 1/2 inches long and 1 3/4 inches internal diameter.

To assemble the parts of the tube and secure their proper alignment, a brass tube (not shown in the figure) is made with an inside diameter of 1 1/4 inches, which will just slip over the ends of the electrodes, and an outside diameter of 1 1/2 inches, which will fit inside the central section of the x-ray tube. This is cut into three or more longitudinal sections to permit removal after the tube is assembled. These sections keep the electrodes in line and at the proper distance, 1/8 inch, from the central section, but permit longitudinal adjustment. The x-ray tube is then placed in a vertical position with the cathode end up. The wax cups at *A* and *C* are partly filled with melted red wax, the glass tubes warmed and pressed into them. When the wax has cooled the x-ray tube is inverted and the wax seals are made at *B* and *D*, taking care that the end of the target comes at the proper place alongside the slit or window with which the central part may be provided and that the end of the cathode focusing tube is exactly 1/8 inch from the target. The wax seal at *E* may be done in a similar manner.

It might appear that separate insulated water supply systems would be necessary but this is easily avoided by bringing the water to and taking it from each electrode through approximately 6 meters of glass tubing having an internal diameter of about 5 mm. Such a tube of water will usually be found to have a resistance of some millions of ohms and hence the loss of energy due to current shunted through the water will be negligible. The electrodes are connected to these glass tubes through several feet of flexible rubber tubing thus allowing them to be withdrawn for cleaning without removing the water connections. The tube will operate most satisfactorily if the central part and the mid point of the high tension winding of the transformer are grounded, in which case the potential between the electrodes and other parts of the apparatus is only one half that between the electrodes themselves. Hence there is much less tendency for a discharge to start to a more distant part, such as the pump, where the distance is perhaps many times the mean free path of the gas molecules and cumulative ionization by collision would readily occur.

Whenever air is admitted to an x-ray tube the parts which have been outgassed by heating adsorb air and this is again driven off when the tube is reevacuated and the filament lighted, causing some delay in obtaining a stable vacuum. This trouble is reduced to a minimum in this tube since all the inner surfaces are kept cool, except the filament and the connecting parts close to it. These heat and are outgassed quickly so that it is usually possible to have the tube in satisfactory operation within five minutes after starting the pump.

With an alternating emf from the high tension side of a transformer a saturation current of 200 milliamperes was reached at 1000 volts. By the use of direct current or rectified alternating current from a system of kenotrons it would be possible to greatly increase the electron current without increasing the applied potential.

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TEMPERATURE CONTROL APPARATUS

By Lorenzo A. Richards

In the pursuance of experimental work of various kinds it is frequently necessary to control the temperature of an air chamber or tank of water. While satisfactory temperature control devices are usually available at supply houses, the cost of sensitive and reliable equipment is generally high and sometimes prohibitive. The following is a description of apparatus which may be easily constructed and which may be used to control the temperature of either an air or a water bath. This equipment, over a period of years, has demonstrated its usefulness for this kind of work and at the present time is being employed in several different research laboratories.

CONSTANT TEMPERATURE AIR BATH

While making some capillary potential measurements in soils* it was found necessary to maintain part of the apparatus at a uniform temperature, somewhat less than room temperature. To do this a temperature case was constructed on one of the laboratory tables. The inside dimensions of the chamber were four by four by seven feet.** The walls were made of compo-board and were about four inches thick, the intervening space being filled with sawdust and wood shavings. Cold tap water was circulated through an automobile radiator which was mounted in front of an electric fan at one end of the case. A small stream of water sufficed to keep the case temperature within two or three degrees of the tap water temperature. Then, in order to keep the temperature of the chamber up to the desired uniform value, an electric heating unit was installed. Nichrome resistance wire, supported by asbestos strips, was strung back and forth in front of the radiator and a thermo-regulator controlled the current through this wire.

The automobile radiator is necessary of course only when below-room temperatures are desired.

* These measurements were made in the Physics Department of the Utah Experiment Station and are included in an article which has been accepted for publication in the Journal Agricultural Research.

** The case described above had a two-by-four foot opening centrally located in one side. This opening was closed by two square window sashes arranged to slide horizontally past each other, on grooved slides. The sashes were installed near the inner side of the thick wall and the opening was closed on the outer side by a compo-board door, hinged at the top and fitting flush with the outer surface of the case. (Celotex or Insulite will provide better heat insulation than compo-board.)

THERMO-REGULATOR

The thermo-regulator is in principle the same as that used by Lovelace and Frazer[1] and incorporates important improvements made by Thomas.[2] Slight changes have been made in the form of the regulator and it has been found more convenient to use other liquids than toluene. Fig. 1 shows how the thermal expansion of the liquid *L*, ether or methyl alcohol, is utilized to make or break the electrical contact between

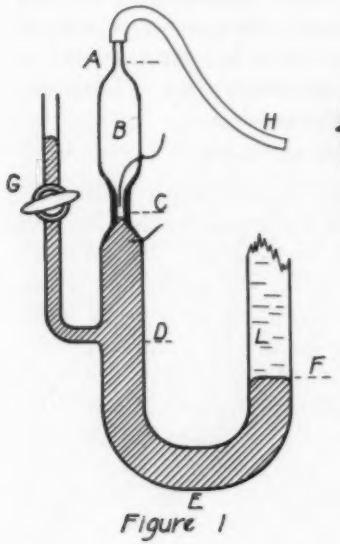

Figure 1

Figure 2

Figure 4

Figure 5

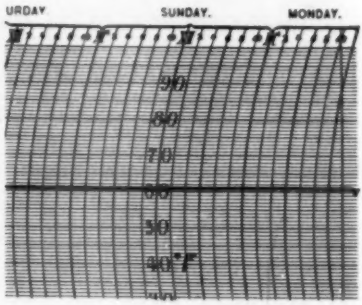

Figure 3

FIGS. 1–5. *Thermo-regulator.*

mercury and the platinum electrode at *C*. The contact points are enclosed in a hydrogen chamber *B* so there is no fouling of the points due to oxidation. The operating temperature is controlled by changing, through the stopcock *G*, the amount of mercury in the regulator.

The glass blowing involved in making the instrument is not difficult and may be accomplished in a few minutes with an ordinary gas blast

lamp. The materials required are a glass stop cock, a piece of ten or twelve millimeter bore soft glass tubing and about five centimeters of 26 guage platinum wire. The large tube is drawn out to a fine capillary at *A* and left open until the regulator is filled. The inside diameter of the tube at the constriction *C* is from one to two millimeters and the distance from *C* to *D* is about five centimeters. The large tube, before bending, should be at least 200 centimeters long. The shape of the coiled part used for the air bath thermo-regulator is shown in Fig. 2.

When the glass blowing is completed the whole tube is filled with the expansion liquid, care being taken to remove all the air or vapor that collects at the bends of the tube. Ether or methyl alcohol may be used. Ether has about a fifty percent greater expansion coefficient than alcohol but its high vapor pressure may cause trouble when the regulator is used at higher temperatures. The hydrogen pressure in the chamber *B*, Fig. 1, must always be great enough to prevent the expansion liquid from boiling or breaking into the vapor phase. This trouble will not appear for temperatures below 35°C for ether or below 75°C for alcohol, but such a possibility should be provided against if higher temperatures are used. When the working temperature will permit, ether should be used because it is easier to remove from the parts of the regulator where it is not wanted.

After the regulator has been filled—let us say with ether—it is held in an upright position and clean mercury is slowly admitted through the stopcock until, with occasional tilting, the mercury fills the tube from F to *D*. The regulator is then clamped in an upright position, a piece of light rubber tubing is attached at *A* and a stream of air is passed into the chamber *B* and out through the stopcock. The glass tube from *E* to *A* is carefully heated with a bunsen flame so as to remove all traces of ether, both above and below the mercury surface. No harm is done if, during this heating process, vapor is caused to form in the tube above *F* because this can later be condensed with cold water.

When the ether is removed, the stopcock is cleaned and lubricated and a stream of hyrogen is substituted for the stream of air. If a hydrogen tank is not available, a zinc-hydrochloric acid generator may be used. The generator should be operated for some time so as to expel the air, and the issuing gas should be passed through a drying tube to remove the water vapor. When the test-tube flame test indicates that the chamber *B* is filled with pure hydrogen, the stopcock is closed and a pinch clamp is placed on the hydrogen tube at *H*. Mercury is

again added through the cock until its surface in the large tube is about a centimeter below the contact point at *C*. This addition of mercury increases the gas pressure in the chamber *B*. It is easier to set the regulator to operate at the desired temperature when the hydrogen is at atmospheric pressure. This pressure may be obtained by releasing, for a short time the pinch clamp at *H*. The capillary at *A* is then sealed off with a pointed flame.

The regulator may well be mounted on a piece of heavy metal screen so as to permit free circulation of the air. The platinum electrodes should be connected by means of copper wire to binding posts that are securely fastened to the mounting. A regulator like the one shown in Fig. 2 was used in the air chamber described above. During an eighteen month period of almost continuous service the instrument never once stuck or failed to operate, and there was no visible oxidation at the contact point. The thermograph shown in Fig. 3 indicates that the temperature fluctuations within the case were too small to be observed with a Julian Friez recording thermometer.

With but minor changes in form this regulator may be employed to control the temperature of a water bath. The type shown in Fig. 4 was made up for a shallow tank used on a laboratory table, while the one in Fig. 5 was designed for use in a deep tank and can be mounted on a narrow board. Where very precise control is desired the ether containing tube may be considerably lengthened and the lower platinum electrode may be insulated so that all of the mercury affecting the regulation may be submerged. However, Thomas[3] found that temperature fluctuations were too small to be observed on a sensitive Beckman thermometer even when this latter precaution was not taken.

A PENCIL-TYPE ELECTRIC HEATER

Electric heaters suitable for water bath temperature control work form an important part of the equipment and it will be found that satisfactory units of any desired heating capacity from 50 to 600 watts are not easy to obtain. The following is a description of a pencil-type electric heater developed by the writer. Units of this kind have been in use for several years and have proved to be well suited for temperature control work. These heaters can be made up having any desired heating capacity and from relatively simple materials. The unit consists of nichrome wire wound on a porcelain tube which is enclosed in a water tight tubular brass case of slightly larger diameter. An

ordinary electric insulator may be used for the porcelain core and the thin walled brass tubing for the jacket may be obtained from any plumbing establishment. The dimensions and structural details are given in Figs. 6, 7, and 8.

Nichrome wire, the size and length being selected to give the desired heating capacity, is wound on the porcelain tube as indicated in Fig. 6. The ends may be anchored by a single turn of soft iron wire. The tube should be smeared over with a thin layer of wet clay—ordinary moulding clay will do—so as to keep the wires evenly spaced after the power is turned on. At *A* and *B*, Fig. 6, short pieces of eighteen gauge nichrome wire are brazed on to the heater element wire. It is very essential that this larger size of heat resisting wire be inserted at this point. The brazing, using brass or spelter with boraz flux, may

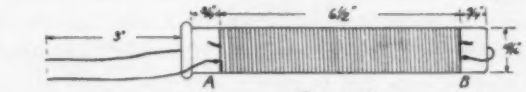

Figure 6

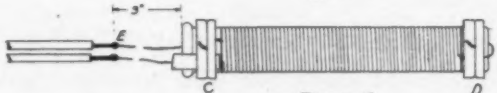

Figure 7

Figure 8

FIGS. 6–8. *Electric Heating Unit.*

be accomplished in a hot blast lamp flame, or better, with acetylene or hydrogen. Then, as shown at *C* and *D* Fig. 7, a spool of thin asbestos sheeting is wound on each end of the tube. This spool should not cover over any current carrying wire of the size used on the heater element proper, and should not be large enough to bind or cause difficulty when inserting the heater element into the brass case. A double layer of the sheeting is placed under the spool at *C* so as to insulate the terminal wire from the case. The eighteen-gauge nichrome wires are then brazed to twelve- or fourteen-gauge insulated copper wires as indicated at *E*.

Fig. 8 shows a section of the completed unit. The lead wires at *F* may be conveniently insulated by wrapping them with the asbestos

sheeting. The jacket for the unit is made by soldering a one-half inch galvanized iron pipe into the brass tube. This joint may be made sufficiently strong by first soldering two copper washers on the iron pipe. These washers should fit the brass tube rather snugly so that when the pipe is inserted the washers give rigidity to the joint. A water tight solder seal is then made at *H*.

After the heater element has been inserted and tested to make sure that the electrical circuit is not grounded to the case, several layers of the asbestos sheeting are fitted in the end of the tube at *I* and a copper disc is soldered in place. The case is then submerged in water and air pressure applied at the top of the pipe to test the case for leaks. If none exist, a strip of cloth is stuffed in around the lead wires at the top of the pipe and hot wax poured in to close the opening.

An advantage of this heater lies in the fact that it can be very easily renewed. It must not, however, be heated up except under water or the solder joints will be melted. If carefully constructed, 500 watt heaters may be expected to give a year and a half of continuous service without trouble.

HEATING-CURRENT RELAY

The regulator described above, like most sensitive regulators, is indirect acting. It closes a circuit when the heater current should be turned off, and hence requires a current relay. This latter device

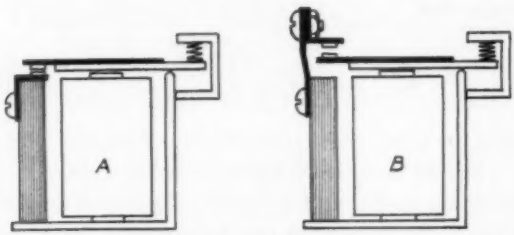

FIG. 9. *Current relay.*

usually adds much uncertainty to the arrangements and requires attention to keep it operating properly, especially if the heating circuit requires a very heavy current.

An inexpensive relay made by the General Electric Company (CR 2810-1265) may without much trouble be adapted for this work and completely overcomes the difficulties ordinarily experienced with heating current relays. Fig. 9, *A*, shows the arrangement of the breaker

points as supplied by the company, and *B* shows what changes are necessary to adapt the relay for use with an indirect acting thermo-regulator. This change can be very easily made. The relay has two sets of breaker points, one on each side, but since one set is all that is required the other pair may be kept for replacement.

This relay works directly from the 110 volt ac line, thus eliminating the danger of over heating due to failure of the relay current and making step down transformers or dc voltage sources unnecessary. It is silent in operation and has but a low power consumption. The large silver contact points have a high current carrying capacity and the positive vigorous action insures good connections. One of these relays, used for over a year in connection with the apparatus described above, required no attention whatever and never once failed to operate.

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PLATINIZING GLASS AND OTHER SUBSTANCES

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The essential steps in the process described herein are not new. In fact, the prepared solution is obtainable commercially. But the process does not appear to be so generally known as its usefulness warrants. Moreover, certain improvements in the method of preparing the solution and in its application are described, and new uses mentioned.

The materials needed for preparing the solution are: Dry Crystalline Pt Cl 4, Oil of Rosemary, and Oil of Lavender flowers.

The procedure is as follows: If the platinic chloride, which is very hygroscopic, is at all moist it should be dried. Since it is very easily reduced by heat to Pt and Cl, a steam bath is safest, but this requires many hours. If sufficient care is taken it may be dried in five or ten minutes in a small crucible over an open gas flame. The moist Pt Cl 4 will liquify when heated. By applying and removing flame as frequently as need be to maintain very gentle boiling, it can be thoroughly dried without reduction, provided a drop, or even less, of HCl is put in occasionally from the end of a stirring rod. The greatest danger of reduction comes with the efforts to drive off the last traces of the water. It should therefore be kept acidulated until the last and the heat removed just before the last pasty mass of chloride (underneath the solid crust) solidifies. Reduction is readily indicated by blackening.

An amount of oil of rosemary, about twice as great by volume as the platinic chloride, is then poured on, and the chloride crushed and mixed with the oil until no lumps at all are left and the resulting mass is black and about the consistency of thick paint. It is then allowed to stand for an hour or longer.

At this point one of two courses may be followed, according to whether the solution is to be used simply for preparing glass to receive solder, or whether the finest flawless mirrors are to be produced.

If only the former is desired, a quantity of oil of lavender, about three times as great by volume as the compound already prepared, is added and thoroughly mixed. The solution is then ready to apply.

In order to explain how the solution is applied, the simplest case may be chosen, that of preparing a glass tube 1/4″ in diameter for receiving a coat of solder.

The tube should be warmed so that the oily solution will spread freely and the (closed) end dipped in the solution, the tube held in a

vertical position and the oil allowed to run off for, say half a minute. the last drop being caught on the edge of the bottle neck. The tube is then turned in various positions so as to spread the film over the surface as uniformly as possible; at the same time passing it back and forth through a gas flame. At a temperature of about 200°C the surface will become glossy and jet black. At a temperature of about 320°C the black coat will change into a brilliant mirror. If a very adhesive coat is desired, the tube is heated still further until the first sign of bending is seen. The tube can then be soldered just as though it were a solid platinum tube, except that the platinum film may be dissolved and removed by the fused solder if the latter is applied too hot or kept fused unduely long.

If the more refined solution is desired, the black mass formed by the oil and Pt Cl 4 and allowed to stand, is poured into the tube of a centrafuge and absolute alcohol added to fill up the tube (the alcohol being first used to wash out the crucible which contained the black compound). After five or ten minutes running the supernatant liquid may be poured off and a mass of pale yellow needle crystals will appear. These crystals may be washed a second or third time in the same way, for they are insoluble in alcohol.

The crystals are then dissolved in oil of lavender, the amount depending on the concentration desired. For mirrors, the proportions will be of the order 25- oil to 1- platinum by weight.

A chemical reaction takes place between the oil of rosemary and the platinic chloride, as evidenced by the evolution of heat and the formation of the crystals. A second exothermic reaction also takes place between the crystals of the platinum compound and the oil of lavender. The final result is the formation of an organic compound of platinum, which is the active principle of the solution.

The solution appears to work better after standing a few days and is just as good after standing for several months in a corked bottle.

Application of the solution: If a film of exceptional perfection is required, such, for example, as for preparing a slit for a spectroscope or making a very fine conducting spiral, it is important that all foreign particles, such as dust, lint, or platinum black, reduced from the chloride, should be removed by filtering through a miniature filter and funnel, using a suction flask but placing a small test tube over the stem of the funnel to catch the solution. A test for foreign particles can be made by applying a platinum coat to glass and burnishing it with a cloth. Pin holes or dark specks will indicate impurities.

The solution prepared as described may now be applied to the surface which it is desired to coat with platinum. A very thin film of the solution will give a brighter mirror than a thicker layer. If the solution is too concentrated (in which case, more oil of lavender should be added) or is applied too heavily, the film will be gray, opalescent, or black, but can be burnished bright. If all conditions are right, a mirror of incomparable brightness will be formed. In order to make the coat adhesive the glass surface must be heated to barely perceptible redness for ordinary glass. Further heating will cause checking of the surface. If the most perfect and best reflecting coat is desired, it should be of such thinness that a bright object, such as a window, may be seen through it. If a thicker coat is desired, any number of thin coats (as many as twenty have been applied) may be put on. The brilliancy is still maintained and the multiple coat behaves as a single film and is as adhesive as the first one applied.

When properly applied, the film is mechanically, an integral part of the glass. To illustrate, a wire soldered to the platinized surface and forcibly pulled loose, will take a chip of glass with it.

Various materials of various shapes may be platinized. The following are some of the materials which will take a mirror coat: Glass, dehydrated borax, quartz, mica bone, calcite sapphire, diamond, or any vitreous material which will not fuse or decompose or react chemically with the platinizing solution at a temperature of 320°C. Metals such as Pt and pure Ni, which do not oxidize or react with the solution at this temperature will take bright coats, which cannot be rubbed off. But, on the vitreous materials, the film will not be strongly adhesive unless the material can be heated until it begins to be plastic. For example, it could never be made adhesive on diamond, because there is no temperature at which diamond becomes plastic. It cannot be made strongly adhesive on quartz, because the Pt film will have evaporated by the time the high temperature at which quartz becomes viscous is reached. The same may be said for unglazed porcelain, though glazed porcelain will take the most tenacious coat. It does not seem possible that a strongly adhesive film could be deposited on a mirror, because the surface of the mirror would be distorted if the temperature of plasticity were reached. Vacuum tight seals to glass bulbs can be made by platinizing the parts and soldering. The solder has the advantage over wax that it can be degased readily.

The best mirrors are produced when the flame is played directly on the surface treated with the solution. If the surface to be platinized

is inaccessible to direct contact with the flame, such, for example, as the inside of a tube, a good mirror will not be formed. In fact, the deposit will not even be good enough to receive solder. But if some arrangement is made to carry either a flame or a stream of hot air into direct contact with such a surface, the film will be as perfect as in the former case. Films of the best quality were deposited inside of a glass tube, 1 mm in diameter, by forcing a stream of hot air into the tube; no heat being applied from the outside.

Films made on flat surfaces of glass by baking in an oven were of poor quality. It appears to be necessary to remove the gases formed by the decomposing solution as rapidly as they are formed. Hence, vigorous blowing of the surface while it is being heated is essential. A vacuum furnace was not tried, neither was a hydrogen furnace tried, nor blowing of the surface while in the furnace.

There are a number of useful "things" which can be done by using this method of depositing platinum.

Very neat resistance units can be made by depositing the film on glass. If the platinized surface is sprayed with a solution of beeswax in benzol and allowed to dry, a steel point will cut a perfectly sharp clean furrow in the platinum film provided it has not been fired on too hard. The firing may be done afterwards if desired. A beautiful helical spiral as fine as a fraction of a mil between turns can be cut on a glass rod. They may be cut bifilar or "non-inductive." An archemedes spiral may be cut on a flat surface; a very fine slit may be made for a spectroscope, a grid for a selenium cell; a small heating coil may be made in the same way.

But the resistance of this form of platinum is unstable, changing as much as five percent or more, and the temperature coefficient of resistance is low, i.e., .0016 as compared with .0038 for pure platinum. No doubt this is due to the formation of a small amount of platinum carbide when the Pt and carbon from the oil are heated together.

It was at first supposed that indefinitely high resistances might be obtained with extremely attenuated films. Investigation proved, however, that a limit is reached at about 1/4 megohm for a centimeter square of film, at which point conductivity ceases abruptly. This is because the molecules composing the film are not uniformly distributed but, as in sputtered films, are laid down in flocks and conductivity ceases at the point where the flocks become islands. Even before this point is reached, the resistance shows great variability and the film is easily ruptured by over voltage. The manner in which this takes place

is very striking. A pinpoint arc will appear at some place on the film. This will begin to migrate and at the same time ramify into unnumerable branches, continuing in this way until the circuit is broken. The microscope then reveals a pattern of strikingly beautiful feathery designs.

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